Cash, Checkable Deposits and Cost of Inflation

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March 2013

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Cash, Checkable Deposits and Cost of Inflation

The welfare cost of inflation is revisited with a search-theoretic model wherein distinct features of cash and checkable deposits as means of payment are incorporated. Our result implies that the presence of an interest-bearing checkable deposit amplifies the cost of inflation significantly via a channel of option value of money.

Put differently, the cost of inflation is substantially underestimated under the assumption of non-interest-bearing M1. This is in stark contrast to the previous studies in which the assumption of non-interest-bearing M1 leads to a considerable overvaluation of an inflation cost. This finding provides the rationale for a low inflation target by the central bank where the payment system is highly developed.

Keywords: cash, interest-bearing checkable deposit, cost of inflation

JEL Classification: E31, E41, E51

I. Introduction

With a great progress in monetary-search theory over the last 20 years, recently search-theoretic models of money have been adopted to assess the welfare cost of inflation (See, for instance, Nosal and Rocheteau (2011, pp. 154-160) for a comprehensive survey). In such models, the benefits of fiat money are spelled out explicitly and hence, the welfare cost of inflation can be understood based on sound micro-foundation. One thing shared by Bailey's (1956) traditional approach and search-theoretic one is that money demand is defined as the aggregate money balance of non-interest-bearing M1. For the case, Rocheteau and Craig (2008) show that the costs of inflation across the two approaches are essentially identical if the terms of trade in search-theoretic models are determined by the buyer's take-it-or-leave-it offer.

But since the 1980s, interest-bearing demand deposits such as NOW account have been steadily expanded and in July 2011, Regulation Q that banned interest payment on demand deposits was eventually repealed. In addition, most households recently use electronic forms of payment, in particular check card associated with the interest-bearing demand deposits. According to Mester (2006), the fraction of households that use check cards had surged from about 18% in 1995 to about 59% in 2004. As illustrated in Figure 1, this is also the case in our economy: i.e., the ratio of the amount of check-card transactions to nominal GDP remained 0.2% in the first quarter of 2004 but it has increased up to 6.1% in the first quarter of 2012.

An interest-bearing demand deposit is cash-like asset in the sense that it is typically insured by the government and provides almost immediate liquidity service. However, cash and demand deposits have quite different features as a means of payment, respectively. Unlike cash, for instance, the demand deposit bears some return but incurs transactional cost such as record-keeping cost. Therefore there is a potential risk of misleading on the cost of inflation when M1 is simply regarded as non-interest-bearing monetary asset. Put in another way, in order to understand monetary-policy effect precisely, we have to disentangle the effect of inflation associated with interest-bearing demand deposits. In our economy, this issue seems to be particularly crucial because the fraction of demand deposits out of M1 exceeds 90%.

Figure 1: Ratio of check card transaction to Nominal GDP

Source: Economic Statistics System, The Bank of Korea.

Building on this point, Bali (2000), Cysne (2003), Jones et al. (2005), and Cysne and Turchick (2010) study the cost of inflation by incorporating interest-bearing demand deposits into a shopping-time or a money-in-utility-function model. The essence of their results is that the cost of inflation is considerably overestimated under the assumption of non-interest-bearing M1. An interest-bearing demand deposit lessens the user-cost of money holdings and hence the decline in the money demand (M1) with inflation will be subdued in the equilibrium, which makes the cost of inflation smaller.

However, these works focused only on the extensive-margin effect (the effect on the money demand). They are silent to the intensive-margin effect (the effect on the terms of trade). Indeed, in the presence of an interest-bearing checkable deposit, the sellers may be more willing to trade to acquire money compared to cash-only economy because money has an “option-like” feature in the sense that it can be held in the form of cash itself or be invested into interest-bearing asset without any loss of liquidity. This option value might be also affected by inflation because the higher prices imply the less opportunities to utilize the option feature of money.

This paper studies the cost of inflation focalizing this intensive-margin effect. In order to do that, as properly pointed out by Lucas (2000), we need a model that embodies distinct roles of cash and interest-bearing demand deposit as means of payment. In this regard, we take notice of a payment pattern observed in the real world such that cash is typically used for small transaction, whereas a check card, a representative means of payment associated with interest-bearing demand deposits, is typically used for relatively large transaction. For example, Bounie and Francois (2009) find from the diary data in France that the average sizes for cash and debit-card transaction are 10.8 Euro and 51.3 Euro, respectively. Internally consistent way, a matching model of money in Shi (1995) and Trejos and Wright (1995) augmented with wealth distribution can materialize these features. In that model, the benefits of money are spelled out explicitly and the amount of transaction as well as the choice of a means of payment are driven by heterogeneous preferences that are generated endogenously by a non-degenerate wealth distribution.¹⁾

More specifically, with the beginning-of-period wealth holdings, each agent chooses a portfolio that consists of cash and checking-account deposit. Then each agent moves into the decentralized market where she is randomly matched with another agent. In a bilateral meeting, the terms of trade are determined by a buyer's take-it-or-leave-it offer. If a buyer in a bilateral trade uses a check card to pay for her consumption-good purchase, it is withdrawn from her account and transferred to the seller immediately, where the seller incurs the associated transaction cost. It is consistent with the real-world observation that check-card providers (usually banks) typically charge transaction fees to retailers (sellers) not to users of check cards. If a buyer uses cash, there is no such cost. This is also consistent with an observation from the real world: i.e., for sellers, transaction cost of accepting check card is much more expensive than cash (see, for example, Humphrey 2004). After pairwise meetings, the balance in checking account is redeemed to each agent with the promised return. There is no return for the early-withdrawn deposit for consumption-good purchases. At the end of a period, lump-sum type of money creation occurs and then an inflation tax is imposed.

1) Alternatively, we might also consider the search-model of money in Lagos and Wright (2005). But we do not think that it is appropriate for our project at least because wealth effect is shut down via a quasi-linear preference.

In order to measure the cost of inflation with the model, we should solve it numerically because a closed-form solution is almost ruled out due to an endogenous non-degenerate distribution. Hence, we first parameterize the model in line with the previous literature and characterize a steady state numerically for a given inflation rate. Then, as usual, the welfare cost of 10% inflation is measured by calculating the compensation consumption that makes the welfare in 10% inflation equivalent to that in 0% inflation.

Our results are striking, in particular compared to the previous literature. First, as mentioned, Bali (2000), Cysne (2003), Jones et al. (2005), and Cysne and Turchick (2010) argued that the presence of interest-bearing demand deposit effectively suppresses the welfare cost of inflation. But our results show that it leads to an increase in the welfare cost of inflation. This consequence is mainly attributed to the option value of money, which is not considered explicitly in the existing literature. Furthermore, the difference in the welfare cost of 10% inflation between money-only and coexistence economy turns out to be considerable. The welfare cost of 10% inflation with checkable deposit is almost 3 times higher than that of money-only economy. It is worth noting that in the coexistence economy, welfare level itself is always higher than that in the money-only economy. That is, as the conventional wisdom, interest-bearing checkable deposits can enhance social welfare but it also may bring up a substantial inflationary distortion. This implies that the central bank should place top priority on the price stability as the usage of means of payment that are backed by interest-bearing demand deposits increases.

The rest of this paper is organized as follows. Section 2 describes the background environment, followed by the equilibrium characterization in Section 3. Section 4 discusses the implications of checkable deposits on the cost of inflation. Section 5 summarizes the paper with a few concluding remarks.

II. Model

The model basically builds on Zhu and Wallace (2007) and Kwon and Lee (2012). Time is discrete and continues forever. There is a $[0, 1]$ continuum of each of $K > 2$ types of infinitely lived agents with K distinct types of specialized good and one general consumption good, where both type-specific and general good are divisible and perishable. A type $k \in \{1, 2, \dots, K\}$ agent produces only good k and consumes only good $k + 1$ (modulo K). General goods are homogeneous and consumed by all agents regardless of specialization types. Notice that as we will see, a type-irrelevant general good is simply a technical device to pay for interest on checking accounts. If an interest is paid by money creation, the support of an endogenous wealth distribution is changed over time, which makes the model complex. A type k agent enjoys per-period utility given by

$$u(q_{k+1}) - q_k + \mathcal{U}(g),$$

where $q_{k+1} \in \mathbb{R}_+$ is consumption of good $k + 1$, $q_k \in \mathbb{R}_+$ is production of good k , $g \in \mathbb{R}_+$ is the consumption of general good, $u'' < 0 < u'$, $\mathcal{U}'' < 0 < \mathcal{U}'$, $u(0) = \mathcal{U}(0) = 0$, $u'(\infty) = \mathcal{U}'(\infty) = 0$, and $u'(0)$ is sufficiently large. Each agent maximizes expected discounted utility with a discount factor $\beta \in (0, 1)$.

There exists indivisible money which is symmetrically distributed across the K specialized types. Let $\bar{m}$ and M denote the exogenous average wealth per type and the exogenous upper bound on individual wealth holdings, respectively. Then the set of possible individual wealth holdings can be denoted by $\mathbf{M} = \{0, 1, \dots, M\}$. Each agent can hold monetary wealth in cash or checkable deposit from which a check card can be used as a means of payment. The intra-temporal information on accounts is kept by the government and she can produce general good using the balance of checkable deposit. Specifically, general good is produced only by the government that has access to a linear technology $G = \theta D$, where G is the quantity of general good produced and D is the balance of deposit after the trades of specialized good.

The sequence of actions in a period is as follows. An agent entering a period with $m \in \mathbf{M}$ chooses a portfolio $\omega = (c, d)$ subject to $c + d \leq m$, where c and d denote cash holdings and checkable deposit, respectively. There is no cost in adjusting port-

folios. After the portfolio is chosen, each agent is randomly matched with another agent. Trades occur only in single-coincidence meetings in which type- k agent meets type- $(k + 1)$ agent. Agents in a meeting know each other's specialization type and portfolio. However, trading histories are private and people cannot commit to their future actions, which makes a medium of exchange essential.

In a single-coincidence meeting, a buyer makes take-it-or-leave-it offer (q, p) , where q denotes quantity of good produced by a seller and p denotes the amount of wealth transferred to a seller. If a buyer uses a check card to pay p , it is withdrawn from the buyer's account and transferred to the seller immediately, where the seller incurs the associated disutility cost $\phi > 0$ per transaction. The cost ϕ can be interpreted as record-keeping cost that is irrelevant to the amount of transaction. If a buyer uses cash, there is no such cost. This is consistent with the survey of Food Marketing Institute: i.e., transaction cost of accepting check card is larger than cash (Humphrey 2004). After pairwise trades, the balance of deposit is redeemed to each agent with θ units of general good per unit of balance. There is no return for the early-withdrawn deposit for specialized-good trades. Then, intra-temporal transaction records of checkable deposit are wiped out completely.

At the end of a period, money creation occurs in a lump-sum manner and then a kind of inflation tax is imposed. Specifically, each agent with $m \in M \setminus \{M\}$ gets a unit of money with probability μ and then each unit of money is confiscated with probability τ , where the latter part of the proportional reduction is a normalization: i.e., for a given μ , τ is determined so that the amount of money created according to μ is completely disintegrated before moving into the next period. This policy is essentially equivalent to the standard lump-sum money creation with divisible money (see, for instance, Lucas and Woodford 1994, Li 1995, and Deviatov and Wallace 2001). Finally, agents go on to the next period with the end-of-period money holdings.

III. Equilibrium

We consider steady-state allocations which are symmetric across specialization types. A symmetric steady state consists of a set of function (v, π, λ) that satisfy the conditions described below. The function $v : M \rightarrow \mathbb{R}$ and $\pi : M \rightarrow [0, 1]$ pertain to the beginning of a period and prior to the choice of portfolios such that $v(m)$ is the expected discounted value of having wealth m and $\pi(m)$ is the fraction of each specialization type with wealth m . The function $\lambda : \Omega \rightarrow [0, 1]$ pertains to the after portfolio choice and before the pairwise meeting such that $\lambda(\omega)$ is the fraction of each specialization type with a portfolio ω , where $\Omega = \{ \omega = (c, d) \in \mathbb{Z}_+^2 : c + d \leq M \}$ denotes the set of feasible individual portfolios.

Following to the sequence of events, we begin with a portfolio-choice stage. Letting $W : \Omega \rightarrow \mathbb{R}$ denote the expected utility after the portfolio choice and before the pairwise meeting, the portfolio choice problem for an agent with m is

$$J(m, W) = \max_{\omega \in \Gamma(m)} W(\omega) \quad (1)$$

where $\Gamma(m)$ denotes the set of feasible portfolios for an agent with m , which can be defined as

$$\Gamma(m) = \{ \omega = (c, d) \in \mathbb{Z}_+^2 : c + d \leq m \}.$$

Let the set of maximizers in (1) be $\mathbb{S}_1(m, W)$. If $\mathbb{S}_1(m, W)$ contains multiple elements, we allow for all possible randomizations over them. This set of randomizations can be expressed as

$$\Delta_1(m, W) = \{ \delta_m : \delta_m(\omega) = 0 \text{ if } \omega \notin \mathbb{S}_1(m, W) \}.$$

Then $\Lambda(W, \pi)$, the set of portfolio distributions on Ω , can be defined as

$$\Lambda(W, \pi) = \{ \lambda : \lambda(\omega) = \sum_m \pi(m) \delta_m(\omega) \text{ for } \delta_m(\omega) \in \Delta_1(m, W) \}. \quad (2)$$

We next turn to pairwise trades. Consider a generic single-coincidence meeting between a buyer with $\omega = (c, d)$ and a seller with $\tilde{\omega} = (\tilde{c}, \tilde{d})$. Let $m_\omega = (c + d)$ and $m_{\tilde{\omega}} = (\tilde{c} + \tilde{d})$ denote the total wealth implied by the portfolio ω and $\tilde{\omega}$, respectively. For the meeting, the set of feasible offers from a buyer to a seller can be defined as

$$\Gamma(\omega, \tilde{\omega}) = \{p : p \in \{0, 1, \dots, \min\{m_\omega, M - m_{\tilde{\omega}}\}\}\}.$$

With a tie-breaking rule by which a seller accepts all offers leaving her no worse off, a buyer's problem can be simplified as

$$\max_{p \in \Gamma(\omega, \tilde{\omega})} = \{u[\tilde{v}(m_\omega + p) - \tilde{v}(m_{\tilde{\omega}}) - \phi 1_{[p > c]}] + \tilde{v}(m_\omega - p) + \mathcal{U}[(d - (p - c)1_{[p > c]})\theta]\} \quad (3)$$

where $\tilde{v} : M \rightarrow \mathbb{R}$ denotes the expected utility after the pairwise meeting and before the money creation, and $1_{[x]} = 1$ if and only if x is true. Let the set of maximizers in (3) be $\mathbb{S}_2(\omega, \tilde{\omega}, \tilde{v})$ and the maximized value of that be $g(\omega, \tilde{\omega}, \tilde{v})$. Noting that the payoff with portfolio $\omega = (c, d)$ as a seller is $\tilde{v}(m_\omega) + \mathcal{U}(\theta d)$, $W(\omega)$ should satisfy

$$W(\omega) = \alpha \sum_{\tilde{\omega}} \lambda(\tilde{\omega}) g(\omega, \tilde{\omega}, \tilde{v}) + (1 - \alpha)[\tilde{v}(m_\omega) + \mathcal{U}(\theta d)] \quad (4)$$

where $\alpha = 1/K$, the probability of a single-coincidence meeting as a buyer.

Now, we can describe the evolution of wealth distribution induced by pairwise trades. As in the portfolio-choice stage, we allow for all possible randomizations over the elements in $\mathbb{S}_2(\omega, \tilde{\omega}, \tilde{v})$. It is convenient to define this set of randomizations over the post-trade wealth of a buyer such that

$$\begin{aligned} \Delta_2(\omega, \tilde{\omega}, \tilde{v}) &= \{ \delta(\cdot; \omega, \tilde{\omega}, \tilde{v}) : \delta(m; \omega, \tilde{\omega}, \tilde{v}) = 0 \\ &\text{if } m \notin \{m_\omega - p(\omega, \tilde{\omega}, \tilde{v})\} \text{ for } p(\omega, \tilde{\omega}, \tilde{v}) \in \mathbb{S}_2(\omega, \tilde{\omega}, \tilde{v}) \}. \end{aligned}$$

Then, the set of post-trade wealth distributions on M can be defined as

$$\begin{aligned} \Phi(\tilde{v}, \lambda) &= \left\{ \varphi \in \mathbb{R}_+^{M+1} : \varphi(m) = \alpha \sum_{(\omega, \tilde{\omega})} \lambda(\omega) \lambda(\tilde{\omega}) [\delta(m; \cdot) + \delta(m_\omega + m_{\tilde{\omega}} - m; \cdot)] \right. \\ &\quad \left. + (1 - 2\alpha) \sum_{\omega} \lambda(\omega) 1_{\{m_\omega = m\}} \text{ for } \delta \in \Delta_2(\omega, \tilde{\omega}, \tilde{v}) \right\}. \quad (5) \end{aligned}$$

The probability measure in the first line of the right-hand side in (5) corresponds to single-coincidence meetings, while the second line corresponds to all other cases.

The evolution of wealth distribution induced by the money creation and confiscation is as follows. Let $\mathfrak{M}$ denote the transition matrix of wealth after the money creation.

Then, the element in row $m \in \mathbf{M}$ and column $m' \in \mathbf{M}$ of $\mathfrak{M}$ can be defined as

$$\mathfrak{M}(m, m') = \begin{cases} 1 - \mu & \text{if } m = m' \\ \mu & \text{if } m = m' - 1 \\ 1 & \text{if } m = m' = \mathbf{M} \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

Similarly, let $\mathfrak{D}$ denote the transition probability after the money confiscation. Then, the element in row $m \in \mathbf{M}$ and column $m' \in \mathbf{M}$ of $\mathfrak{D}$ can be defined as

$$\mathfrak{D}(m, m') = \begin{cases} \binom{m}{m'} \tau^{m-m'} (1-\tau)^{m'} & \text{if } m \geq m' \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

Because a proportional confiscation is nothing but a normalization, τ should satisfy

$$\sum_{m=0}^{M-1} \mu \varphi(m) = \sum_{m'} \sum_m \varphi(m) \mathfrak{m}_{(m, m')} m' - \sum_{m''} \sum_{m'} \sum_m \varphi(m) \mathfrak{m}_{(m, m')} \mathfrak{D}_{(m', m'')} m'' \quad (8)$$

where the left-hand side represents the amount of money created and the right-hand side represents the amount of money confiscated. Then the value of holding $\hat{m} \in \mathbf{M}$ after pairwise meetings and before money creation can be expressed as

$$W(\hat{m}) = \beta(1-\mu) \sum_m \mathfrak{D}_{(\hat{m}, m)} v(m) + \beta\mu \sum_m \mathfrak{D}_{(\hat{m}+1, m)} v(m) \quad (9)$$

where the discount factor β appears because agents carry the balance of money after confiscation to the next period. Finally, the set of distribution on $\mathbf{M}$ after the money creation and confiscation can be written as

$$\Pi(v, \lambda) = \left\{ \pi : \pi(m) = \sum_{m'} \sum_{m''} \varphi(m') \mathfrak{m}_{(m', m'')} \mathfrak{D}_{(m'', m)} \text{ for } \varphi(m') \in \Phi(W, \lambda) \right\} \quad (10)$$

where the dependence of Π on v is through the dependence of Φ on W and then W on v as in (9). Now, we can define a steady state as follows.

Definition 1 *A symmetric steady state is a set of function (v, π, λ) such that*

- *the value function v must satisfy $v(m) = J(m, W)$, where $J(m, W)$ is given by (1) and $W : \Omega \rightarrow \mathbb{R}$ is given by (4);*
- *the probability measure π of wealth distribution must satisfy $\pi \in \Pi(v, \lambda)$, where $\Pi(v, \lambda)$ is given by (10);*
- *the probability measure λ of portfolio distribution must satisfy $\lambda \in \Lambda(W, \pi)$, where $\Lambda(W, \pi)$ is given by (2).*

The existence of a steady state for some parameters is a straightforward extension of the result in Lee et al. (2005), and Zhu and Wallace (2007); if $\bar{m}$ and $M/\bar{m}$ are large enough, and (ϕ, θ, μ) are sufficiently close to zero, respectively, then there exists a steady state (v, π, λ) with v strictly increasing, strictly concave, and π having full support.

IV. Comparisons of Inflation Costs

With a given steady state (v, π, λ) for an inflation rate i , the expected lifetime utility of a representative agent prior to the assignment of wealth according to π can be expressed as

$$\mathbb{W}_i = \left(\frac{\alpha}{1 - \beta} \right) \pi U \pi' + \left(\frac{1 - \alpha}{1 - \beta} \right) \pi \mathcal{U}_d$$

where the element in row $j \in M$ and column $k \in M$ of the matrix U is

$$u[q(j, k)] - \tilde{q}(j, k) + \mathcal{U}\{\theta[d_j - (p(j, k) - c_j)1_{|p(j, k) > c_j|}]\}$$

with $\tilde{q}(j, k) = q(j, k) + \phi 1_{\{p(j, k) > c_j\}}$ and the j -th component of the $(M+1)$ vector $\mathbf{u}_d$ is $\mathcal{U}(\theta d_j)$. As usual, the welfare cost of 10% inflation can be measured by asking how much additional consumption should be given to buyers in every single-coincidence meeting to bring $\mathbb{W}_{10}$ up to $\mathbb{W}_0$. Specifically, we first find an additive consumption compensation Δ that solves

$$\mathbb{W}_0 = \left(\frac{\alpha}{1 - \beta} \right) [\pi(\mathbf{U}_\Delta) \pi']_{10} + \left(\frac{1 - \alpha}{1 - \beta} \right) [\pi \mathbf{u}_d]_{10}$$

where the element in row $j \in M$ and column $k \in M$ of $\mathbf{U}_\Delta$ is

$$u[q(j, k) + \Delta] - \tilde{q}(j, k)$$

and $[\xi]_{10}$ denotes ξ in the steady state of 10% inflation. Then, the welfare cost of 10% inflation is expressed as a ratio of Δ to the average consumption.

We are now ready to compute and compare the cost of inflation between money-only and coexistence economies. Hereinafter our discussion mainly draws on the observations of numerical examples because the endogeneity of non-degenerate wealth distribution rules out closed-form solution.

4.1. Numerical Environment

We first parameterize the model as follows. We set $K = 3$ which is the smallest number of types eliminating the possibility of double-coincidence of wants in pairwise meeting. We set $(M, \bar{m}) = (3\bar{m}, 20)$ so that the bound on money holdings and the indivisibility of money are not too severe. If the indivisibility of money is too severe in this type of a model, almost all monetary offers are in the set of $\{0, 1\}$. However, as we can see in Table 1 and 3, it is not the case in our numerical examples. Furthermore, as we can see in Figure 5 and 6 in Appendix, $Z = 3\bar{m}$ is large enough in the sense that almost no one is at the upper bound in a steady state and hence the result would be hardly affected even if a larger M were imposed.

We next assume $\beta = (0.96)^{1/4}$ which corresponds to quarterly model period with a standard annual discount factor of 0.96. Together with $\bar{m} = 20$ and a quarterly model period, $\mu = 0.50$ implies 10% of an annualized growth rate of money. We then set the return rate of checkable deposit (θ) as 0.025%, which is very close to an annual real

return rate of MZM deposits reported in Šustek (2010). And we set ϕ as 0.15% of q^* = $\text{argmax}[u(q) - q]$, which is slightly lower than the estimated cost (0.2% of GDP) incurred by the U.S. banks in providing demand-deposit services in the early 1990s (Aiyagari et al. 1998).

Finally, we let $u(q) = q^{1-\eta}/(1-\eta)$ and $u(g) = \ln(1+g)$, where η is chosen to fit the model to the data about the share of the number of check-card transactions. Based on the grocery-store data, Klee (2008) reports that out of cash and checkable-deposit relevant transactions, the latter accounts for around 38.6% in terms of the number of transactions. In our parameterized version of model, $\eta = 0.62$ generates the ratio of 38.4% for the inflation rate of 3%.

In order to check the plausibility of overall parameterization above, we consider the case $\theta = 0$ for which the portfolios that consist entirely of cash are obviously equilibrium portfolios. This equilibrium corresponds to existing money-only models. Table 1 reports summary statistics of the cash-only steady states with the parameter values above other than θ . In the table, $\mathbb{E}(q)$, $\mathbb{E}(p)$, and $\mathbb{W}$ denote average consumption over single-coincidence meetings, average monetary offer over single-coincidence meetings, and welfare, respectively. Notice that the cost of 10% inflation is almost the midpoint of the range of existing estimates (1 ~ 1.5%) in the context of search-theoretic models with the buyer-take-all bargaining solution.²⁾ This immediately implies that our numerical environment is not out of the ordinary and indeed, it is in line with the previous literature of money-only models.

Table 1: Summary statistics for money-only steady states

	$\mathbb{E}(q)$	$\mathbb{E}(p)$	$\mathbb{W}$	Welfare cost
0% inflation	0.9123	1.3888	19.8968	–
3% inflation	0.8644	2.4637	19.7491	0.4581
10% inflation	0.6754	5.4800	19.5680	1.2127

Note: This table shows average consumption ($\mathbb{E}(q)$), average monetary offer ($\mathbb{E}(p)$), and welfare ($\mathbb{W}$) at the cash-only steady state under either 0%, 3%, or 10% inflation.

2) See Nosal and Rocheteau (2011, pp. 154-160) for a comprehensive survey of the existing estimates of the welfare cost of inflation. Craig and Rocheteau (2008) also show that these estimates are essentially consistent with the estimates based on the traditional methodology of Bailey (1956).

In addition, Table 2 reports payment pattern by wealth levels in the steady state with 3% inflation and $(K, \bar{m}, M, \beta, \eta, \theta, \phi) = (3, 20, 60, 0.99, 0.62, 0.00025, 0.0015)$. The payment patterns contrast starkly across the agents with different wealth. That is, sufficiently poor buyers typically use cash to pay for their relatively small quantity of consumption-good purchases, whereas sufficiently rich buyers typically use check card to pay for their relatively large quantity of consumption-good purchases. This seems to be consistent with the cross-sectional feature of payment pattern observed in the real world. Bounie and Francois (2009) find from the diary data in France that the average sizes for cash and debit-card transaction are 10.8 Euro and 51.3 Euro, respectively.

From the point of our economy, the parameterization above is also quite reasonable. First, noting that the check-card usage has been increasing steadily in these days, the ratio of the amount of check-card transactions to M1 in our economy was about 4.3% in the first quarter of 2012, whereas in the steady state of our parameterized version of model with 3% inflation, the ratio is around 4.8%. Moreover, the ratio of cash holdings to $\bar{m}$ in the steady state of a parameterized version of model is 6.7%, whereas it is 6.9%

Table 2: Payment patterns by wealth levels

	Average offer		
		cash fraction	check-card fraction
$m \leq 5$	0.9971	1.0000	0.0000
$6 \leq m \leq 15$	1.6879	0.9692	0.0308
$16 \leq m \leq 20$	2.0675	0.9312	0.0688
$21 \leq m \leq 25$	2.5137	0.3132	0.6868
$m \geq 26$	3.0844	0.0000	1.0000

Note: This table show payment patterns by wealth levels in the steady state with 3% inflation and parameters set as $(K, \bar{m}, M, \beta, \eta, \theta, \phi) = (3, 20, 60, 0.99, 0.62, 0.00025, 0.0015)$.

in our economy during the period 2004-2011.

4.2. Checkable Deposit and Cost of Inflation

Table 3 reports summary statistics of steady states for cash-checkable deposit co-existence economy and money-only economy ($\theta = 0$). In the table, $\mathbb{E}(q)$, $\mathbb{E}(p)$, and $\mathcal{F}$ denote average consumption, average monetary offer and the share of the number of check-card transactions, respectively. The welfare in the cash-checkable deposit coexistence economy is higher than that in money-only economy because money can be invested into interest-bearing deposits.

The most striking result is that the cost of 10% inflation is more than threefold as large as that of money-only economy. One of the sources of this large difference stems from foregone return of checkable deposit. The amount of wealth involved in the trades of specialized good [$\mathbb{E}(p)$] increases with inflation and hence the balance of deposit measured at maturity declines (0% inflation: 18.9552 $\rightarrow$ 3% inflation: 18.3403 $\rightarrow$ 10% inflation: 18.0723). As a consequence, as reported in Table 4, welfare gain from return on deposits declines respectively by 0.0153 and 0.0221 in 3% and 10% inflation compared to 0% inflation, where the welfare gain from return on

Table 3: Summary statistics for steady states

	inflation	$\mathbb{E}(q)$	$\mathbb{E}(p)$	$\mathcal{F}$	$\mathbb{W}$	Welfare cost
cash & checkable deposit	0%	1.0518	1.0517	0.0273	21.0937	–
	3%	0.8535	2.2741	0.3844	20.7252	1.1881
	10%	0.6878	5.3395	0.9239	20.0631	3.7920
cash only	0%	0.9123	1.3888	–	19.8968	–
	3%	0.8644	2.4637	–	19.7491	0.4581
	10%	0.6754	5.4800	–	19.5680	1.2127

Note: This table shows average consumption ($\mathbb{E}(q)$), average monetary offer ($\mathbb{E}(p)$), share of the number of check-card ($\mathcal{F}$), and welfare ($\mathbb{W}$) for cash-checkable deposit coexistence economy, and money-only economy under either 0%, 3%, or 10% inflation.

deposits is calculated by

$$\left(\frac{\alpha}{1-\beta} \right) \sum_{(j,k)} \pi(j)\pi(k) \mathcal{U}[\theta[d_j - (p(j,k) - c_j)1_{\{p(j,k) > c_j\}}]] + \left(\frac{1-\alpha}{1-\beta} \right) \sum_j \pi(j) \mathcal{U}(\theta d_j).$$

Furthermore, as inflation goes up, $\mathbb{E}(p)$ increases but agents try to economize on their cash balances as much as possible. Therefore, the ratio of the average cash holdings to the average monetary offer declines: i.e., 0.9847 (0% inflation) $\rightarrow$ 0.5894 (3% inflation) $\rightarrow$ 0.0412 (10% inflation). This implies that the share of the number of check-card transactions increases with inflation and consequently the consumption declines due to transaction cost of check card (ϕ). The welfare loss due to this change in payment pattern increases respectively by 0.0178 and 0.0528 in 3% and 10% inflation compared to 0% inflation (see Table 4), where the welfare loss due to ϕ is calculated by

$$\left(\frac{\alpha}{1 - \beta} \right) \sum_{(j, k)} \pi(j) \pi(k) \{u[\tilde{q}(j, k)] - u[q(j, k)]\}.$$

Interestingly, however, welfare loss of inflation due to foregone return and transaction cost of check card accounts for less than one tenth of total loss of welfare.

Table 4: Welfare loss from return and cost of check card

	0% inf.	3% inf.	10% inf.
Welfare gain from return on deposits	0.4727	0.4574	0.4506
changes from 0% (A)	(-)	(-0.0153)	(-0.0221)
Welfare loss due to ϕ	0.0010	0.0188	0.0538
changes from 0% (B)	(-)	(0.0178)	(0.0528)
Welfare loss due to θ and ϕ (B-A)	-	0.0331	0.0749
Welfare-level changes from 0%			
coexistence	(-)	(-0.3685)	(-1.0306)
money-only	(-)	(-0.1477)	(-0.3288)

Note: This table shows the size of welfare gains from return on deposits and welfare losses due to the transaction cost of check card.

That is, as shown in Table 4, welfare in the coexistence economy decreases relatively rapidly with inflation compared to money-only economy and this discrepancy cannot be explained by foregone return from deposits and transaction cost of check card.

Then what causes such a big difference between the two? Figure 2 shows the marginal valuation of money ($v(i) - v(i - 1)$) as a function of wealth level.

When the inflation rate is 10%, its difference between money-only and coexistence economy is minimal. However, when the inflation rate is 0%, marginal valuation of money is considerably high in the coexistence economy compared to money-only economy. This stark difference results from the *option value of money* which works only in the coexistence economy. As an almost perfect substitute for money to pay for consumption purchase, checkable deposit carries the option feature which yields positive return if held to maturity. Hence sellers in the decentralized market are willing to transfer more output in exchange for a unit of money. Table 5 reports the average quantity of specialized good that the sellers are willing to produce in exchange for a unit of money. Its difference between money-only and coexistence

Figure 2: Marginal valuation of wealth

Note: This graph plots the marginal valuation of money for the two different economies under either 0%, or 10% inflation.

economy is minimal when inflation rate is 10%, whereas it is substantial when inflation rate is 0%.

This option-value-of-money channel is fully effective in a 0% inflation economy

because there is no inflation-tax burden for the money acquired via pairwise trades. However, the channel is eroded with inflation because inflation tax is imposed for the money obtained in the decentralized market. A high enough inflation makes the channel debilitated sufficiently and hence equilibrium allocations in the coexistence economy become very similar to those in the money-only economy (see Table 1 and 3).

Table 5: Sellers' willingness to produce per unit of money

	0% inflation	3% inflation	10% inflation
Cash only	0.6574	0.3569	0.1314
Cash & Checkable deposits	1.0066	0.3837	0.1374

Note: This table shows the average quantity of specialized good that the sellers are willing to produce in exchange for a unit of money for cash-checkable deposit coexistence economy, and money-only economy under either 0%, 3%, or 10% inflation.

Finally, we check the sensitivity of the inflation cost to the change in the return rate of checkable deposit (θ) and the cost of check-card transaction (ϕ). Excepting (θ, ϕ) , we use the parameter values discussed in Section 4.1: i.e., $(K, \bar{m}, M, \beta, \eta) = (3, 20, 60, 0.99, 0.62)$. Table 6 reports the welfare cost of 10% inflation when θ (or ϕ) is changed by 10% from the benchmark $\theta = 2.5 \times 10^{-4}$ (or $\phi = 1.5 \times 10^{-3}$). Not surprisingly, the higher θ (or ϕ) makes the cost of inflation larger but there is no considerable change in the magnitudes of the cost.

Table 6: Welfare cost

	$\phi = 1.5 \times 10^{-3}$ $\theta = 2.5 \times 10^{-4}$	$\tilde{\phi} = \phi$		$\tilde{\theta} = \theta$	
		$\tilde{\theta} = 0.9\theta$	$\tilde{\theta} = 1.1\theta$	$\tilde{\phi} = 0.9\phi$	$\tilde{\phi} = 1.1\phi$
Welfare cost	3.7920	3.7719	3.8296	3.7897	3.8203

Note: This table shows the size of welfare cost under 10% inflation for other θ and ϕ .

V. Concluding Remarks

In this paper, we have explored the effects of interest-bearing checkable deposit on the welfare cost of inflation by using a standard search-theoretic model of money. Our model implies that welfare cost of inflation is substantially underestimated if we simply consider M1 as non-interest-bearing asset. This is in stark contrast to the previous studies such as Bali (2000), Cysne (2003), Jones et al. (2005), and Cysne and Turchick (2010), in which the assumption of non-interest-bearing M1 leads to a considerable overestimation problem of the inflation cost.

Interestingly, the consumption distortion turns out to be the main component of the inflation distortion. This channel of distortion is salient in the model with an interest-bearing checkable deposit where the option value of money works. Meanwhile, the welfare loss induced by foregone return and transaction cost of check cards turns out to be relatively small. However, we cannot say definitely that they are not important in understanding the true cost of inflation. This is because besides a liquid asset, there is illiquid asset whose rate of return is higher than that of a liquid asset due to the liquidity premium at least. If we introduce illiquid asset explicitly, then the welfare loss due to foregone return might come into a greater prominence. In order to explore this effect, we need to construct a model in which other than fiat money and liquid asset, illiquid asset is demanded under the plausible cost and return structures.

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Appendix: Steady State

Figure 3: Value function: money-only equilibrium

Note: These graphs show the value function at different wealth levels for the money-only economy under either 0%, 3%, or 10% inflation respectively.

Figure 4: Value function: coexistence equilibrium

Note: These graphs show the value function at different wealth levels for the coexistence economy under either 0%, 3%, or 10% inflation respectively.

Figure 5: Distribution function: money-only equilibrium

Note: These graphs plot the density function of cash holdings for the money-only economy under either 0%, 3%, or 10% inflation respectively.

Figure 6: Distribution function: coexistence equilibrium

Note: These graphs plot the density function of cash holdings for the coexistence economy under either 0%, 3%, or 10% inflation respectively.

Table 7: Optimal portfolio: 0% inflation

z	c	d	z	c	d	z	c	d
1	1	0	21	1	20	41	0	41
2	1	1	22	1	21	42	0	42
3	1	2	23	1	22	43	0	43
4	1	3	24	1	23	44	0	44
5	1	4	25	1	24	45	0	45
6	1	5	26	1	25	46	0	46
7	1	6	27	1	26	47	0	47
8	1	7	28	1	27	48	0	48
9	1	8	29	2	27	49	0	49
10	1	9	30	2	28	50	0	50
11	1	10	31	2	29	51	0	51
12	1	11	32	2	30	52	0	52
13	1	12	33	2	31	53	0	53
14	1	13	34	2	32	54	0	54
15	1	14	35	2	33	55	0	55
16	1	15	36	2	34	56	0	56
17	1	16	37	2	35	57	0	57
18	1	17	38	2	36	58	0	58
19	1	18	39	0	39	59	0	59
20	1	19	40	0	40	60	0	60

Note: This table shows the optimal portfolio consisting of cash and checkable deposits by wealth levels under 0% inflation.

Table 8: Optimal portfolio: 3% inflation

z	c	d	z	c	d	z	c	d
1	1	0	21	2	19	41	0	41
2	1	1	22	2	20	42	0	42
3	1	2	23	0	23	43	0	43
4	1	3	24	0	24	44	0	44
5	1	4	25	0	25	45	0	45
6	1	5	26	0	26	46	0	46
7	1	6	27	0	27	47	0	47
8	1	7	28	0	28	48	0	48
9	1	8	29	0	29	49	0	49
10	1	9	30	0	30	50	0	50
11	1	10	31	0	31	51	0	51
12	2	10	32	0	32	52	0	52
13	2	11	33	0	33	53	0	53
14	2	12	34	0	34	54	0	54
15	2	13	35	0	35	55	0	55
16	2	14	36	0	36	56	0	56
17	2	15	37	0	37	57	0	57
18	2	16	38	0	38	58	0	58
19	2	17	39	0	39	59	0	59
20	2	18	40	0	40	60	0	60

Note: This table shows the optimal portfolio consisting of cash and checkable deposits by wealth levels under 3% inflation.

Table 9: Optimal portfolio: 10% inflation

z	c	d	z	c	d	z	c	d
1	1	0	21	0	21	41	0	41
2	1	1	22	0	22	42	0	42
3	2	1	23	0	23	43	0	43
4	2	2	24	0	24	44	0	44
5	2	3	25	0	25	45	0	45
6	2	4	26	0	26	46	0	46
7	3	4	27	0	27	47	0	47
8	3	5	28	0	28	48	0	48
9	3	6	29	0	29	49	0	49
10	0	10	30	0	30	50	0	50
11	0	11	31	0	31	51	0	51
12	0	12	32	0	32	52	0	52
13	0	13	33	0	33	53	0	53
14	0	14	34	0	34	54	0	54
15	0	15	35	0	35	55	0	55
16	0	16	36	0	36	56	0	56
17	0	17	37	0	37	57	0	57
18	0	18	38	0	38	58	0	58
19	0	19	39	0	39	59	0	59
20	0	20	40	0	40	60	0	60

Note: This table shows the optimal portfolio consisting of cash and checkable deposits by wealth levels under 10% inflation.

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