Forecasting the Term Structure of Government Bond Yields Using Credit Spreads and Structural Breaks

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In this paper, we investigate whether credit spread curve information helps forecast the government bond yield curve and whether the joint dynamics of the government bond yields and credit spreads have structural changes. For this purpose, we use a joint dynamic Nelson-Siegel (DNS) model of the term structures of U.S. Treasury interest rates and credit spreads. We find that this joint model produces substantially more accurate out-of-sample Treasury yields forecasts compared with a standard DNS yield curve only model. We also find that the predictive gain from incorporating the credit spread curve information substantially increases if the joint model accounts for structural changes in the dynamics of yield and credit spread curves. In addition, our model incorporates a zero lower bound restriction ensuring that our predictions are economically plausible.

**Keywords:** Out-of-sample forecasting, term structure, credit spread, Nelson-Siegel model, Bayesian MCMC estimation

**JEL Classification:** C11; C53; E43; E47
I. Introduction

Many studies (e.g., Longstaff and Schwartz (1995), Duffee (1998), Collin-Dufresne et al. (2001), Davies (2008)) find the negative relationship between risk-free interest rates and credit spreads. Longstaff and Schwartz (1995) theoretically explain this negative relationship by the model-implied positive relationship between the firm value and the risk-free rate. An increase in the firm value reduces the probability of default and in turn increases the corporate bond price. Consequently, the corporate bond yield falls and therefore the credit spread tightens. Thus, in theory, the government bond yields and the credit spread should be interconnected. In this paper, we examine if information contained in the term structure of credit spreads helps predict government bond yields. For this purpose, we follow Christensen and Lopez (2012) and extend a dynamic Nelson-Siegel (DNS) approach, proposed by Diebold and Li (2006), to jointly model the dynamics of the government bond yield curve and the credit spread curve.

Our proposed model has five factors to describe the dynamics of the government bond yield curve and the credit spread curve. In this model, the dynamics of the government bond yield curve is modeled by three latent factors defined by the DNS model. In the DNS model, these factors are interpreted as the level, slope, and curvature of the yield curve. The credit spread curve is modeled by two factors that are interpreted as the level and slope of the credit spread curve. These two additional factors have factor loadings with functional forms similar to those for the level and slope factors of the DNS model. Our analysis shows that two factors explain most of the variations in the credit spreads of different maturities. To model the interactions between the yield curve and the credit spread curve, we allow all factor shocks to be correlated. Our modeling specification with correlated shocks to the yield curve factors and the credit spread curve factors is motivated by the relationships between the yield curve and the credit spreads with the macroeconomic environment reported in the literature (e.g., Ang et al. (2006), Hamilton and Kim (2002), and Gilchrist and Zakrajsek (2012)).

With aim to improve the predictive ability of the above described model, we
incorporate two additional extensions to the model. First, we incorporate possibility of multiple change-points into the joint process of the five model factors. This extension is motivated by findings in the literature that the U.S. Treasury yield curve underwent structural breaks in mean and volatility over the past few decades (e.g., Chib and Kang (2013), Dai et al. (2007), Bech and Lengwiler (2012), and Ang et al. (2008)). Some of these structural breaks might be associated with changes in the business cycle. The credit risk is also closely related to the variations in the business cycle. Therefore, the relationship between government bond markets and corporate bond markets possibly changes over time. Indeed, by isolating so called deflationary and inflationary regimes, Davies (2008) find that the credit spread is substantially more sensitive to the changes in the risk-free rate in the deflationary regime compared to the normal or inflationary regime. Our second extension is a zero lower bound restriction imposed on the yields and credit spreads. In our model specification, following Kang (2013), the joint distribution of the bond yields and credit spreads is normal and truncated to non-negative values. This restriction ensures that forecasted nominal short-term bond yields are positive. In our analysis, we show that this extension of the model is critical in producing economically reasonable yield forecasts in the low interest rate environment.

Our results show that the joint model of the yield and credit spread curves has better out-of-sample forecasts than the yield curve only model. In particular, the joint model produces more accurate forecasts of all maturity yields and at all horizons up to 12 month ahead than the standard three-factor DNS yield curve model. In addition, incorporating structural changes to the joint dynamics of the Treasury yields and the credit spreads substantially improves predictive accuracy of the model. Specifically, the predictive gain is maximized when two change-points are incorporated. We estimate those change-points occurred in March 2008 and January 2010 and appear to be associated with the period of the recent financial crisis. The timing of the first structural change is closely associated with the decline in the short-term risk-free interest rate and the dramatic widening of credit spreads. The second structural change reflects tightening of credit spreads as the crisis ends. Consistently with our out-of-sample results, in-sample estimation
results show that shocks to the dynamics of the yield factors and shocks to the dynamics of the credit spread factors are strongly correlated and these correlations change over time. Finally, imposing a zero lower bound constraint turns out to be binding to the yield curve, not to the credit spread curve.

Our study is closely related to the work of Christensen and Lopez (2012). The authors propose and estimate a model of joint dynamics of the Treasury yield and credit spread curves in an arbitrage-free dynamic Nelson-Siegel model framework. Similar to our study, they find that adding information in the credit spread improves out-of-sample forecasts of the Treasury yields relative to the performance of the arbitrage-free dynamic Nelson-Siegel model of the yield curve only, proposed in Christensen et al. (2011). Meanwhile our study has several important distinctions from that paper. First, we show that the relationship between the yield curve and the credit spread curve has structural changes and accounting for these changes substantially improves forecast performance. Given our focus to study potential structural changes in the relationship between the Treasury yield curve and the credit spread curve, we propose to use a more parsimonious joint Nelson-Siegel model compared to the arbitrage-free version of the model. While the arbitrage-free model has the theoretical advantage, incorporating potential change-points into a standard Nelson-Siegel model is a more parsimonious extension compared to the arbitrage-free Nelson-Siegel model.

The second distinction is in the credit spread data used in two studies. Christensen and Lopez (2012) consider credit spreads of four business sectors separately. Presumably forecasts of the government bond yields vary for the models based on credit spreads of different sectors.¹ In our study, we use the data on the term structure of the BBB-rated-indexed credit spread. By using the aggregated index we aim at reducing the impact of the industry-specific idiosyncratic noise on forecast results. The choice of BBB spreads is motivated by relatively high liquidity of these corporate bonds and high sensitivity of yields on these bonds to the risk-free rate changes reported in the literature (e.g., Davies (2008)).

Third, we model interactions between the Treasury yields and credit spreads

¹ Christensen and Lopez (2012) report out-of-sample forecasts for the banking sector only.
through correlations of factor shocks. This approach contrast Christensen and Lopez (2012) who account for these interactions by modeling the credit spread curve using four factors: the level and slope factors of the yield curve and the level and slope factors of the credit spread curve. Our analysis suggests that two factors explain most of the variations in the credit spreads in our choice of the credit spread data. Therefore, we use two credit spread factors only to model the credit spread curve. Fourth, we impose a zero lower bound restriction for the yields and it appears to be binding in the low interest rate environment.

The rest of the paper is organized as follows. Section 2 describes the data and motivates the relationship between the default-free bond yields and the credit spreads. Section 3 describes the model. Section 4 explains our Bayesian MCMC estimation method. Section 5 discusses the estimation results and findings. Section 5 concludes.

II. Data and Motivation

Our data comprises the end of month yields on government bonds and the BBB-credit-rated corporate bond index for the period August 1996 through March 2013. The data are obtained from the Federal reserve economic data and the Barclays POINT. The set of the government bonds is 3, 6, 12, 24, 36, 60, 84, and 120 month maturity bonds. The set of the credit spread maturities comprises 24, 36, 60, 84, and 120 months. The term structure of credit spreads is defined by differences between the government bond yields and the corporate bond yields of corresponding maturities. Usually, the corporate bonds have maturities 24 month and longer, therefore the term structure of credit spreads has 24 months as the shortest maturity. As we discuss in the introduction, we pick the BBB spreads because of high sensitivity of yields on these bonds to the risk-free rate changes reported in the literature (e.g., Davies (2008)).

<Figure 1> plots the time series of the government bond yield curve and credit spread curve. This figure show the large variation in the shapes of the yield and credit spread curves over time. To demonstrate co-movement of the yield and
<Figure 1> Term Structure of Government Bond Yields and Credit Spreads

Note: The figure plots the monthly time series of the government bond yield curve and the time series of the credit spread curve. The set of the maturities for the yield curve includes 3, 6, 12, 24, 36, 60, 84, and 120 months. The credit spreads are the differences between the yields of the government bonds and BBB-rated corporate bonds with the 24-, 36-, 60-, 84-, and 120-month maturity. The sample period is from August 1996 to March 2013.
credit spread in the middle of the maturity structures, <Figure 2> displays the time series of the 5-year government bond yield and the 5-year credit spread. From the figure one can see that the relationship between the credit spread and government yield is different during three time periods: the period prior to the recent crisis till mid-2007, the recent financial crisis period from mid-2007 through the end of 2009, and the post-crisis period from 2010. During the pre-crisis period the credit spread level was low and the level of long-term government yields was high. The correlation between the government bond yield and the credit spread was negative 0.22.

During the recent financial crisis the yield curve level decreased, the credit spread drastically widened, and the negative correlation between the government yield and the credit spread increased to -0.72. During this period the credit spread widened in response to increase in credit and liquidity risk and flight-to-quality. Since 2010 the credit spread and the government yield level fell dramatically. However, the credit spread remains more volatile compared to the pre-crisis period. The negative correlation between the government yield and credit spread decreased to -0.47, but remains at a higher level than the correlation during the pre-crisis period. This data analysis suggests that the credit spreads and the

<Figure 2> Treasury Rate and Credit Spread of 5-year Maturity

Note: The figure plots the monthly time series of the 5-year government bond yield and the 5-year credit spread. The sample period is from August 1996 through March 2013.
government yields are correlated, and therefore the information contained in the credit spread might be useful for predicting government yields. However, it appears that the relationship between the government yields and the credit spreads might have structural changes and presumably varies with changes in the macroeconomic environment.

### III. Model

#### 1. The Joint Model of Government Bond Yields and Credit Spreads

In this subsection, first we describe the model for the term structure of government bond yields, second the model for the term structure of credit spreads, and finally the approach we use to model interactions between the government bond yields and credit spreads. To model the government bond yields we use the three-factor DNS model proposed by Diebold and Li (2006). The authors show that this model has a good in-sample fit and out-of-sample forecasts of the yield curve. Below we describe the DNS model.

Diebold and Li (2006) demonstrate that the entire yield curve can be described by three factors that can be interpreted as the level, slope, and curvature of the yield curve. Since yields observed in data are not fully explained by the three factors, empirical implementation of the DNS model adds a maturity-specific measurement error to each yield. The equation below describes the empirical DNS model:

\[
    r_t(\tau) = f_t^L + \frac{1 - e^{-\lambda \tau}}{\lambda \tau} f_t^S + \left[\frac{(1 - e^{-\lambda \tau})}{\lambda \tau} - e^{\lambda \tau}\right] f_t^C + e_t(\tau)
\]

where \(r_t(\tau)\) denotes the government bond yield of \(\tau\)-month maturity at time \(t\) and \(f_t^L\), \(f_t^S\), and \(f_t^C\) denote the time-varying level, slope, and curvature factors, respectively. We note that while \(f_t^S\) denotes the slope factor, the negative of the
slope factor should be associated with the yield curve slope. \( \lambda \) is a parameter of the model that helps fit yields at different maturities. Conditioned on the factors and the measurement error variance, \( r_t(\tau) \) is assumed to be normally distributed with 
\[
e_t(\tau) \sigma_{\varepsilon, \tau}^2 \sim i.i.d. N(0, \sigma_{\varepsilon, \tau}^2).
\]

For our credit spread data, the principal component analysis shows that the first two principal components explains 99.1\% percent of their total variation. Christensen and Lopez (2012) show that the first two principal components can be interpreted as the level and slope of the credit spread curve. Following Krishnan et al. (2007) and Christensen and Lopez (2012) we employ the Nelson-Siegel structure to model the credit spread using two factors as

\[
s_t(\tau) = c_t(\tau) - r_t(\tau)
= x_t^L + \left(1 - \frac{e^{-\tau \lambda_x}}{\tau \lambda_x}\right) x_t^S + v_t(\tau), \quad v_t(\tau) \sim i.i.d. N(0, \sigma_v^2).
\]

where \( s_t(\tau) \) denotes the credit spread between the corporate bond yield \( c_t(\tau) \) of \( \tau \)-period maturity at time \( t \) and the default-free yield \( r_t(\tau) \). \( x_t^L \) and \( x_t^S \) are the credit spread level and slope factors, respectively. \( v_t(\tau) \) denotes the measurement error.

In contrast to Krishnan et al. (2007), we do not add the curvature factor to the credit spread model because in our study the credit spread curve does not have very short maturities and therefore this factor tends to be estimated with low precision. Also unlike Christensen and Lopez (2012), we do not add the government bond yield factors to the above model the credit spread curve. According to the principal component analysis, The first two factors should be sufficient to explain most of the variations in yields. The credit spread model has \( \lambda_x \) parameter that is different from the yield curve parameter \( \lambda \). It allows the models to fit curvatures of two curves independently from each other.

Now we specify the joint process of the yield curve and the credit spread curve in the matrix notation. We start by introducing additional notations. The vector of the yield curve factors and the vector of the credit spread are denoted by
\[ f_t = (f_t^L, f_t^S, f_t^C)' \] and \[ x_t = (x_t^L, x_t^S)' \], respectively. All factors are denoted jointly by \[ \beta_t = (f_t', x_t')' \]. The vectors of factor loadings for the government bond yields and the credit spread we denote by

\[
A_f(\tau) = [1 \ (1 - e^{-\tau \lambda})/\tau \lambda \ (1 - e^{-\tau \lambda})/\tau \lambda - e^{-\tau \lambda}],
\]

\[
A_x(\tau) = [1 \ (1 - e^{-\tau \lambda})/\tau \lambda_x],
\]

Combining the government yields and the credit spreads of all maturities at time \( t \) in one vector \( y_t \), the joint dynamics of government bond yields and credit spread can be expressed by

\[
y_t = A \beta_t + E_t
\]

where

\[
y_t = [r_t(\tau_1) \ r_t(\tau_2) \ \cdots \ r_t(\tau_N) \ s_t(\tau_1^*) \ s_t(\tau_2^*) \ \cdots \ s_t(\tau_N^*)]',
\]

\[
\{\tau_i\}_{i=1,2,...,N} = \{3, 6, 12, 24, 36, 60, 84, 120\},
\]

\[
\{\tau_i^*\}_{i=1,2,...,N^*} = \{24, 36, 60, 84, 120\},
\]

\[
A = \begin{bmatrix} A_f & 0_{N \times 2} \\ 0_{N^* \times 3} & A_x \end{bmatrix},
\]

\[
A_f = [A_f(\tau_1)' \ A_f(\tau_2)' \ \cdots \ A_f(\tau_N)']/',
\]

\[
A_x = [A_x(\tau_1^*)' \ A_x(\tau_2^*)' \ \cdots \ A_x(\tau_N^*)']/',
\]

\[
E_t = [e_t(\tau_1) \ e_t(\tau_2) \ \cdots \ e_t(\tau_N) \ v_t(\tau_1) \ v_t(\tau_2) \ \cdots \ v_t(\tau_N^*)] \quad \text{and}
\]

\[
E_t|\Sigma \sim \text{i.i.d.}N(0, \Sigma).
\]

The variance-covariance matrix of the measurement errors is denoted by \( \Sigma \) and it is diagonal. To ensure non-negative government bond yields and credit spreads we assume that the distribution of \( y_t \) conditioned on \( (\beta_t, \Sigma) \) is truncated normal on the interval \([0, \infty)\):

\[
y_t|\beta_t, \Sigma \sim \text{i.i.d.}TN_{[0, \infty)}(A \beta_t, \Sigma) \quad \text{for} \ t = 1, 2, ..., T
\]
This restriction implies that all elements in $y_t$ are non-negative.

Now we specify a stochastic process for the unobserved latent factors $\beta_t$ which describe the dynamics of the yield and credit spread curves. We assume that $\beta_t$ follows a first-order vector autoregressive process:

$$
\beta_t = \mu + G\beta_{t-1} + \epsilon_t, \quad \epsilon_t | \Omega \sim i.i.d. N(0, \Omega)
$$

where $\mu$ is a $5 \times 1$ vector, $G$ is a $5 \times 5$ diagonal matrix, and $\Omega$ is a variance-covariance matrix of the factor shocks $\epsilon_t$.

We model interactions between the government bond yields and credit spreads by allowing instantaneous shocks to all factors to be correlated. In other words, we assume $\Omega$ to be a complete matrix. This assumption differs our modeling approach from Christensen and Lopez (2012) who constrained the variance-covariance matrix of factor shocks to a diagonal matrix. The correlation between shocks to the yield factors and the credit spread factors should help the model pick the interactions between government bond markets and corporate bond markets. Our estimation results confirm economic and statistical significance of these correlations.

In our model specification, we assume the autoregressive $G$ matrix to be diagonal. Diebold and Li (2006) and Christensen et al. (2011) find that the model with independent factors performs out-of-sample forecasts better than the model with factor interactions. Our preliminary empirical experiments also agree with this finding in the literature. Specifically, when the $G$ matrix is unrestricted and the dynamic interaction between all five factors is allowed, the predictive accuracy worsen compared with the case when the $G$ matrix is diagonal.  

2) Factor Process with Change-points

Chib and Kang (2013) finds that the yield curve underwent multiple structural breaks over the past few decades. Therefore, we also allow for changes in the

2) For conciseness of the paper we do not report these results. These results are available upon request.
parameters of the factor process at unknown time points. The break process is modeled through a restricted Markov process $q_t$ such that given $q_{t-1} = j$,

$$q_t = \begin{cases} j & \text{with prob. of } p_{j,j} \\ j+1 & \text{with prob. of } 1 - p_{j,j} \end{cases}$$ (7)

where $q_0 = 1$, $p_{j,j} = \Pr[q_t = j | q_{t-1} = j] \in [0,1]$ for $j = 1, 2, \ldots, J$, and $p_{J+1, J+1} = 1$. $q_t$ can never take a value less than $j$ or bigger than $j+1$. The current regime $j$ will stay at the current value or move to $(j+1)$ in the next period. Once the regime $(J+1)$ is reached, this absorbing regime continues. Note that $(J+1)$ is the number of regimes and the maximum number of structural changes is $J$. We assume that the regime shifts are independent of the measurement errors and the yield curve and credit spread factor shocks conditioned on the past regimes. Given the regime $q_t$ at time $t$ the factors are determined as

$$\beta_t = \mu + G_{\beta} \beta_{t-1} + \epsilon_t, \quad \epsilon_t|\Omega_{\beta} \sim i.i.d. N(0, \Omega_{\beta})$$ (8)

The regime changes in the variance-covariance matrix of the factor shocks may capture not only changes in the factor volatility, but also time-varying relationship between the government bond market and the corporate bond market observed in data. For example, the previous literature (e.g., Ang et al. (2006) and Hamilton and Kim (2002)) finds that the level or slope factors of the yield curve tend to reflect the current stage of the business cycle. Also, the default risk is strongly related to the business cycle and it causes changes in the level of credit spread curve (e.g., Gilchrist and Zakrajsek (2012)). Therefore, the shocks to the yields and credit spreads may be correlated and their variances might be time-varying.

3. Alternative Model Specifications

We consider several alternative model specifications for out-of-sample forecasting to find out (i) if incorporating credit spread information improves forecasts of
<Table 1> Alternative Model Specifications

<table>
<thead>
<tr>
<th>Model</th>
<th>Default-free bond yields</th>
<th>Credit spreads</th>
<th>Number of change-points</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNS–CS(0)</td>
<td>Yes</td>
<td>Yes</td>
<td>0</td>
</tr>
<tr>
<td>DNS–CS(1)</td>
<td>Yes</td>
<td>Yes</td>
<td>1</td>
</tr>
<tr>
<td>DNS–CS(2)</td>
<td>Yes</td>
<td>Yes</td>
<td>2</td>
</tr>
<tr>
<td>DNS(0)</td>
<td>Yes</td>
<td>–</td>
<td>0</td>
</tr>
<tr>
<td>DNS(1)</td>
<td>Yes</td>
<td>–</td>
<td>1</td>
</tr>
<tr>
<td>DNS(2)</td>
<td>Yes</td>
<td>–</td>
<td>2</td>
</tr>
<tr>
<td>CS(0)</td>
<td>–</td>
<td>Yes</td>
<td>0</td>
</tr>
<tr>
<td>CS(1)</td>
<td>–</td>
<td>Yes</td>
<td>1</td>
</tr>
<tr>
<td>CS(2)</td>
<td>–</td>
<td>Yes</td>
<td>2</td>
</tr>
</tbody>
</table>

Note: DNS-CS denotes models with default-free bond yields and credit spreads. DNS denotes models with default-free bond yields only. CS denotes models with credit spreads only. The numbers in parentheses indicate the number of incorporated change-points.

government bond yields, (ii) if incorporating structural changes in the model helps improve forecasts, and (iii) how many structural breaks, if any, the data support. <Table 1> describes alternative models that we consider in our study. DNS(J) denotes the standard three-factor DNS model of the default-free bond yields only. It allows for change-points in the factor process up to $J$ times. $J=0$ indicates that a model does not incorporate any change-points. CS(J) denotes the two-factor model of the credit spreads with $J$ change-points. Finally, DNS-CS(J) is the five-factor model of the joint dynamics of the yield and credit spread curves with $J$ change-points. It uses information on both yield curve factors and credit spread factors in fitting and forecasting. For completeness of our study we also consider the credit spread only model for forecasting credit spreads.

IV. Econometric Methodology

We estimate the model parameters using the Bayesian econometric approach. In this section, we describe our prior for the model parameters, then our empirical MCMC method of sampling posterior distributions, and finally, the criterion used to measure out-of-sample forecasting performance.
1. Prior

Let \( g_{ \theta } \) denote a vector of the diagonal elements of \( G_\theta \). Then the set of the model parameters is given by \( \Theta = \{ P, \gamma, \Omega, \Sigma \} \), where \( P = \{ p_{j,j} \}_{j=1,2,...,J} , \)
\( \gamma = \{ \mu_{q,j}, g_{q,j} \}_{j=1,2,...,J+1} , \)
\( \Omega = \{ \Omega_{q,q} = j \}_{j=1,2,...,J+1} \). We complete model specification by specifying the prior distributions for the model parameters.

We assume priors for \( \mu_{q,j} \) and \( G_\theta \) to be normally distributed. Following the stationarity assumption in the standard DNS model, we constrain the factor process to be stationary through the prior on autoregressive coefficients. The prior of the variance-covariance matrix \( \Omega_\theta \) has an inverse-Wishart distribution. The prior for the measurement error variance is an inverse gamma distribution. The transition probabilities have a beta prior. Given other parameters and \( y_0 \) the prior for \( x_0 \) has a normal distribution with unconditional mean \( ( \bar{\beta}_0 ) \) and variance \( ( \bar{V}_{x_0} ) \) of \( \beta_1 \). We let \( \Sigma_k \) denote the \((k,k)\) element of \( \Sigma \). For \( q_0 = 1,2,...,J+1, \)
\( j = 1,2,...,J, \) and \( k = 1,2,...,(N+N^*), \) the prior distributions for \( \Theta \) and \( \beta_0 \) are summarized as

\[
\begin{align*}
\mu_\theta & \sim i.i.d. N(\bar{\mu}, \bar{V}_\mu), \\
g_\theta & \sim i.i.d. N(\bar{g}, \bar{V}_g), \\
\Omega_\theta & \sim i.i.d. IW(k_0, R_0), \\
\Sigma_k & \sim i.i.d. IG(\nu_0/2, \delta_0/2), \\
p_{j,j} & \sim i.i.d. \beta(a_0, b_0), \\
\beta_0 & \sim N(\bar{\beta}_0, \bar{V}_{\beta_0})
\end{align*}
\]

where

\[
\begin{align*}
\bar{\mu} &= (2, 0.5, 0.5, 1, 0)', \\
\bar{V}_\mu &= 25 \times I_5, \\
\bar{g} &= (0.95, 0.95, 0.95, 0.95, 0.95)', \\
\bar{V}_g &= 0.25 \times I_5, \\
k_0 &= 2, \\
R_0 &= 5 \times I_5, \\
\nu_0 &= 27, \\
\delta_0 &= 1.3, \\
a_0 &= 7.020, \\
b_0 &= 0.143.
\end{align*}
\]

2. Posterior Simulation Method

This subsection describes the method we use to simulate the joint posterior distribution. Let \( Y = \{ y_1, y_2, ..., y_n \} \), \( B = \{ \beta_0, \beta_1, ..., \beta_n \} \), and \( Q = \{ q_1, q_2, ..., q_n \} \). The
joint posterior distribution $\pi(\Theta, B, Q, Y)$, which is the target distribution, is proportional to the product of complete likelihood density $f(Y|B, Q, \Theta)$, the prior density for the latent factors $p(B|\Theta, Q)$ the prior density for the regimes $p(Q|\Theta)$ and the prior density for the parameters ($\pi(\Theta)$). The prior for the regimes is already given by the change-point process. Given the regimes and parameters, the vector of factors $\beta_t$ is determined through the process given in Equation (7). The parameters in $\Theta$ are mutually independent, so $\pi(\Theta)$ is given by

$$
\pi(\Theta) = \prod_{q_t=1}^{J+1} N(\mu_{q_t} | \mu, \Sigma) \times \prod_{q_t=1}^{J+1} N(g_{q_t} | \bar{g}, \Sigma_g) I( | g_{q_t} | < 1 ) 
\times \prod_{q_t=1}^{J+1} JW(\Omega_{q_t} | k_0, R_0) \times \prod_{i=1}^{N+N'} IG(\Sigma_i | \nu_0/2, \delta_0/2) \times \prod_{j=1}^{J} \beta(p_{j,\cdot}|a_0, b_0)
$$

(10)

where $I(\cdot)$ is an indicator function.

We simulate the joint posterior distribution of $(B, Q, \Theta)$ using a MCMC simulation method. Each of the MCMC iterations consists of four stages. After initializing the regime vector $Q$ and factors $B$, we first draw the parameters from $f(\Theta|Y, B, Q)$ based on the M-H algorithm. Then conditional on $(Y, Q, \Theta)$ we draw from $f(B|Y, Q, \Theta)$ by applying the M-H method again. Note that because of the non-normality assumption the multi-move Gibbs sampling algorithm of Carter and Kohn (1994) is not applicable. Therefore, we use the multi-move method proposed in Chib (1998) to generate the regime vector $Q$ conditioned on $(Y, B, \Theta)$. Finally, we sample the posterior predictive draw of the yield curve and credit spread curve. We use the empirical distribution of 5,000 draws from the conditional posterior densities beyond 1,000 burn-in cycles.

3. Forecast Performance Criterion

We follow Zantedeschi et al. (2011) and evaluate the forecasting performance of each of the considered models based on the posterior predictive criterion (Gelfand and Ghosh (1998)). The posterior predictive criterion for the $h$-month-ahead posterior predictive density of the $\tau$-month bond yield and credit spread is
denoted by \( PPC_r(\tau, h) \) and \( PPC_c(\tau, h) \), respectively. They are computed as

\[
PPC_r(\tau, h) = D_r(\tau, h) + W_r(\tau, h)
\]

and

\[
PPC_c(\tau, h) = D_c(\tau, h) + W_c(\tau, h)
\]

where

\[
D_r(\tau, h) = \text{Var}(r_{n+h}(\tau)|Y),
\]
\[
D_c(\tau, h) = \text{Var}(c_{n+h}(\tau)|Y),
\]
\[
W_r(\tau, h) = \left[ r^O_{n+h}(\tau) - E(r_{n+h}(\tau)|Y) \right]^2,
\]
\[
W_c(\tau, h) = \left[ c^O_{n+h}(\tau) - E(c_{n+h}(\tau)|Y) \right]^2.
\]

\((r^O_{n+h}(\tau), c^O_{n+h}(\tau))\) denote the realized \(\tau\)-month government bond yield and credit spread at time \(n+h\) and \((r_{n+h}(\tau), c_{n+h}(\tau))\) denote the corresponding predictive government bond yield and credit spread obtained from the posterior simulation. By definition, a smaller PPC indicates a better forecasting performance.

To see the forecasting performance across maturities and forecast horizons, we compute and report the PPC averaged over the maturities (\(PPC_r(\tau)\) and \(PPC_c(\tau)\)) and the PPC averaged over the forecast horizons (\(PPC_r(h)\) and \(PPC_c(h)\)). That is,

\[
PPC_r(\tau) = \frac{1}{12} \sum_{h=1}^{12} PPC_r(\tau, h),
\]
\[
PPC_c(\tau) = \frac{1}{12} \sum_{h=1}^{12} PPC_c(\tau, h),
\]
\[
PPC_r(h) = \frac{1}{N} \sum_{i=1}^{N} PPC_r(\tau_i, h),
\]
\[
\text{and } PPC_c(h) = \frac{1}{N} \sum_{i=1}^{N} PPC_c(\tau_i, h).
\]
V. Results

In this section, we report out-of-sample forecasting results, discuss the roles of the auxiliary information in the corporate bond market and the structural breaks in forecasting the yield curve, and analyze sources of forecast improvements.

1. Out-of-sample Prediction

To evaluate forecast performance of models using PPC, we conduct recursive out-of-sample predictions. Following Diebold and Li (2006), we consider forecast horizons from one-month to twelve-month ahead. The first set of one-month through twelve-month-ahead forecasts, which is for the period from May 2011 to April 2012, are obtained from estimation of the models based on the data from August 1996 to April 2011. Then, the in-sample data is extended by one month to August 1996 though May 2011, the models are re-estimated, and out-of-sample forecasts for the period June 2011 to May 2012 are obtained. This procedure is repeated until the last out-of-sample forecasts for the period from April 2012 to March 2013 are obtained. PPC is calculated based on 12 forecasts for each horizon generated by the above recursive procedure. <Table 2> and <Table 3> report PPC for all considered models by each horizon and each maturity, respectively. In addition to the models listed in <Table 1>, we report predictive performance of the simple random walk and AR(1) models, denoted RW and AR in the tables. The RW and AR(1) models are a commonly used benchmark model for predicting highly persistent variables.

To analyze the role of the credit spread information in forecasting the government bond yields we compare the models with and without credit spread information and the same number of change points. Specifically, we compare a DNS-CS(J) model with a DNS(J) model, where J is the same for both models. The values of PPC from the models with credit spread information are substantially lower than those from the yield curve model only for all horizons and maturities. These results imply that incorporating instantaneous interaction among shocks to
## $\text{Table 2}$ $\text{PPC}(h)$ for the Government Bond Yields

<table>
<thead>
<tr>
<th>$h$</th>
<th>DNS-CS($0$)</th>
<th>DNS-CS($1$)</th>
<th>DNS-CS($2$)</th>
<th>DNS($0$)</th>
<th>DNS($1$)</th>
<th>DNS($2$)</th>
<th>RW</th>
<th>AR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.297</td>
<td>0.248</td>
<td><strong>0.227</strong></td>
<td>0.290</td>
<td>0.228</td>
<td>0.231</td>
<td>0.272</td>
<td>0.289</td>
</tr>
<tr>
<td>2</td>
<td>0.415</td>
<td>0.357</td>
<td><strong>0.311</strong></td>
<td>0.411</td>
<td>0.319</td>
<td>0.324</td>
<td>0.396</td>
<td>0.428</td>
</tr>
<tr>
<td>3</td>
<td>0.515</td>
<td>0.461</td>
<td><strong>0.383</strong></td>
<td>0.514</td>
<td>0.398</td>
<td>0.401</td>
<td>0.498</td>
<td>0.545</td>
</tr>
<tr>
<td>4</td>
<td>0.596</td>
<td>0.545</td>
<td><strong>0.437</strong></td>
<td>0.599</td>
<td>0.460</td>
<td>0.463</td>
<td>0.583</td>
<td>0.642</td>
</tr>
<tr>
<td>5</td>
<td>0.658</td>
<td>0.607</td>
<td><strong>0.474</strong></td>
<td>0.661</td>
<td>0.504</td>
<td>0.506</td>
<td>0.646</td>
<td>0.717</td>
</tr>
<tr>
<td>6</td>
<td>0.711</td>
<td>0.659</td>
<td><strong>0.498</strong></td>
<td>0.716</td>
<td>0.536</td>
<td>0.539</td>
<td>0.699</td>
<td>0.779</td>
</tr>
<tr>
<td>7</td>
<td>0.747</td>
<td>0.697</td>
<td><strong>0.513</strong></td>
<td>0.756</td>
<td>0.556</td>
<td>0.557</td>
<td>0.735</td>
<td>0.825</td>
</tr>
<tr>
<td>8</td>
<td>0.787</td>
<td>0.742</td>
<td><strong>0.527</strong></td>
<td>0.795</td>
<td>0.577</td>
<td>0.580</td>
<td>0.769</td>
<td>0.870</td>
</tr>
<tr>
<td>9</td>
<td>0.833</td>
<td>0.790</td>
<td><strong>0.552</strong></td>
<td>0.840</td>
<td>0.605</td>
<td>0.607</td>
<td>0.806</td>
<td>0.923</td>
</tr>
<tr>
<td>10</td>
<td>0.875</td>
<td>0.836</td>
<td><strong>0.574</strong></td>
<td>0.879</td>
<td>0.630</td>
<td>0.631</td>
<td>0.840</td>
<td>0.973</td>
</tr>
<tr>
<td>11</td>
<td>0.925</td>
<td>0.888</td>
<td><strong>0.598</strong></td>
<td>0.928</td>
<td>0.657</td>
<td>0.660</td>
<td>0.881</td>
<td>1.033</td>
</tr>
<tr>
<td>12</td>
<td>0.975</td>
<td>0.939</td>
<td><strong>0.622</strong></td>
<td>0.976</td>
<td>0.686</td>
<td>0.690</td>
<td>0.925</td>
<td>1.092</td>
</tr>
</tbody>
</table>

Note: $\text{PPC}(h)$ Bold numbers indicate the lowest PPC for each horizon. RW is the random-walk model and AR is the first-order autoregressive model. In the RW and AR models, the time series are assumed to be mutually independent.

the yield curve and credit spread factors substantially improves government bond yields predictions.

Now we analyze whether incorporating structural changes into the model improves forecasting performance of the models. The DNS-CS(2) model is found to have the best performance in forecasting government yields. As $\text{Table 2}$ and

## $\text{Table 3}$ $\text{PPC}(\tau)$ for the Government Bond Yields

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>DNS-CS($0$)</th>
<th>DNS-CS($1$)</th>
<th>DNS-CS($2$)</th>
<th>DNS($0$)</th>
<th>DNS($1$)</th>
<th>DNS($2$)</th>
<th>RW</th>
<th>AR</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.570</td>
<td>0.552</td>
<td><strong>0.208</strong></td>
<td>0.586</td>
<td>0.316</td>
<td>0.284</td>
<td>0.558</td>
<td>0.652</td>
</tr>
<tr>
<td>6</td>
<td>0.477</td>
<td>0.440</td>
<td><strong>0.066</strong></td>
<td>0.491</td>
<td>0.232</td>
<td>0.228</td>
<td>0.492</td>
<td>0.586</td>
</tr>
<tr>
<td>12</td>
<td>0.465</td>
<td>0.393</td>
<td><strong>0.125</strong></td>
<td>0.476</td>
<td>0.255</td>
<td>0.255</td>
<td>0.531</td>
<td>0.564</td>
</tr>
<tr>
<td>24</td>
<td>0.591</td>
<td>0.469</td>
<td><strong>0.290</strong></td>
<td>0.597</td>
<td>0.385</td>
<td>0.389</td>
<td>0.556</td>
<td>0.661</td>
</tr>
<tr>
<td>36</td>
<td>0.755</td>
<td>0.636</td>
<td><strong>0.540</strong></td>
<td>0.754</td>
<td>0.551</td>
<td>0.587</td>
<td>0.664</td>
<td>0.781</td>
</tr>
<tr>
<td>60</td>
<td>0.906</td>
<td>0.863</td>
<td><strong>0.821</strong></td>
<td>0.898</td>
<td>0.760</td>
<td>0.784</td>
<td>0.828</td>
<td>0.951</td>
</tr>
<tr>
<td>84</td>
<td>0.908</td>
<td>0.917</td>
<td><strong>0.884</strong></td>
<td>0.902</td>
<td>0.802</td>
<td>0.804</td>
<td>0.872</td>
<td>0.930</td>
</tr>
<tr>
<td>120</td>
<td>0.883</td>
<td>0.911</td>
<td><strong>0.878</strong></td>
<td>0.873</td>
<td>0.803</td>
<td><strong>0.795</strong></td>
<td>0.865</td>
<td>0.952</td>
</tr>
</tbody>
</table>

Note: $\text{PPC}(\tau)$ Bold numbers indicate the lowest PPC for each maturity. RW is the random-walk model and AR is the first-order autoregressive model. In the RW and AR models, the time series are assumed to be mutually independent.
Tables 3 report, this model produces the lowest PPC values at all forecast horizons and maturities, except the three-month bond yield for which DNS-CS(1) has a better result. In general, the DNS-CS(2) model forecasts yields better than the DNS-CS(1) and DNS-CS(0) models and the DNS(2) model forecasts yields better than the DNS(1) and DNS(0) models. These results confirm that the yield curve dynamic underwent through structural changes. The results also suggests that the benefit from using the information contained in the credit spreads increases when two change-points are allowed. Our preliminary analysis with three change-points (i.e., DNS-CS(3)) produced forecasts which are almost identical to those of the DNS-CS(2) model. We note that we use only model parameters estimated for the last regime to sample predictive draws.

Table 4 reports the posterior estimates of the change-point dates from the considered models. While the estimated change point dates from the DNS-CS(1) and DNS(1) models are close to each other, the DNS-CS(2) model produces change-point dates different from the DNS(2) models. Also, the change-point dates from the DNS-CS(2) are relatively close to those from the CS(2) model. These results indicate that dynamics of the credit spreads and changes in the relationship between the government yields and credit spreads impact the estimation of the change points in the joint model. The last change-point period from the DNS-CS(2) model is January 2010 and from the DNS(2) model is May 2009. Therefore, relative to the DNS(2) model, the DNS-CS(2) model utilizes a shorter sample of data for the model parameter estimation in regime 3. Despite that, the DNS-CS(2) model forecasts yields better than the DNS(2) model. This observation confirms that the credit spread curve information helps improve government yields predictions.

For completeness of our study we also analyze performance of the models and the role of structural breaks in predicting credit spreads. Table 5 and Table 6 report PPC for credit spreads for each considered horizon and maturity, respectively. Incorporating two change-points in the joint model and the credit spread only model substantially improves predictive performance of the models. It can be explained by substantially different dynamics of the credit spreads during the recent financial crisis. The credit spreads dynamics in this period is modeled by
the parameters in regime 2 between two change-points. Overall, the joint DNS-CS(2) models perform forecast of credit spreads better than the credit spread CS(2) model for the most of horizons and maturities, except the 1-month horizon and 120-month maturity for which the CS(2) model has better results.

<Table 4> Change-points Estimates

<table>
<thead>
<tr>
<th>Model</th>
<th>Change-points</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNS-CS(1)</td>
<td>2009:M1</td>
</tr>
<tr>
<td>DNS-CS(2)</td>
<td>2008:M3, 2010:M1</td>
</tr>
<tr>
<td>DNS(1)</td>
<td>2008:M12</td>
</tr>
<tr>
<td>DNS(2)</td>
<td>1999:M9, 2009:M5</td>
</tr>
<tr>
<td>CS(1)</td>
<td>2008:M1</td>
</tr>
<tr>
<td>CS(2)</td>
<td>2008:M7, 2010:M3</td>
</tr>
</tbody>
</table>

Note: DNS-CS denotes models with default-free bond yields and credit spreads. DNS denotes models with default-free bond yields only. CS denotes models with credit spreads only. The numbers in parentheses indicate the number of incorporated change-points.

<Table 5> \( PPC(h) \) for the Credit Spreads

<table>
<thead>
<tr>
<th>( h )</th>
<th>DNS-CS(0)</th>
<th>DNS-CS(1)</th>
<th>DNS-CS(2)</th>
<th>CS(0)</th>
<th>CS(1)</th>
<th>CS(2)</th>
<th>RW</th>
<th>AR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.606</td>
<td>0.476</td>
<td>0.454</td>
<td>0.896</td>
<td>1.013</td>
<td>0.447</td>
<td>0.542</td>
<td>0.528</td>
</tr>
<tr>
<td>2</td>
<td>0.736</td>
<td>0.559</td>
<td><strong>0.513</strong></td>
<td>0.928</td>
<td>1.302</td>
<td>0.516</td>
<td>0.722</td>
<td>0.694</td>
</tr>
<tr>
<td>3</td>
<td>0.824</td>
<td>0.633</td>
<td><strong>0.568</strong></td>
<td>0.907</td>
<td>1.505</td>
<td>0.580</td>
<td>0.855</td>
<td>0.808</td>
</tr>
<tr>
<td>4</td>
<td>0.910</td>
<td>0.702</td>
<td><strong>0.619</strong></td>
<td>0.896</td>
<td>1.688</td>
<td>0.639</td>
<td>0.984</td>
<td>0.920</td>
</tr>
<tr>
<td>5</td>
<td>0.950</td>
<td>0.733</td>
<td><strong>0.636</strong></td>
<td>0.916</td>
<td>1.814</td>
<td>0.665</td>
<td>1.070</td>
<td>0.980</td>
</tr>
<tr>
<td>6</td>
<td>0.991</td>
<td>0.761</td>
<td><strong>0.657</strong></td>
<td>0.925</td>
<td>1.908</td>
<td>0.697</td>
<td>1.145</td>
<td>1.033</td>
</tr>
<tr>
<td>7</td>
<td>1.019</td>
<td>0.777</td>
<td><strong>0.676</strong></td>
<td>0.939</td>
<td>1.998</td>
<td>0.717</td>
<td>1.217</td>
<td>1.086</td>
</tr>
<tr>
<td>8</td>
<td>1.048</td>
<td>0.796</td>
<td><strong>0.682</strong></td>
<td>0.959</td>
<td>2.072</td>
<td>0.733</td>
<td>1.272</td>
<td>1.126</td>
</tr>
<tr>
<td>9</td>
<td>1.066</td>
<td>0.814</td>
<td><strong>0.692</strong></td>
<td>0.977</td>
<td>2.135</td>
<td>0.748</td>
<td>1.321</td>
<td>1.153</td>
</tr>
<tr>
<td>10</td>
<td>1.081</td>
<td>0.827</td>
<td><strong>0.707</strong></td>
<td>0.980</td>
<td>2.186</td>
<td>0.765</td>
<td>1.371</td>
<td>1.179</td>
</tr>
<tr>
<td>11</td>
<td>1.094</td>
<td>0.852</td>
<td><strong>0.715</strong></td>
<td>1.003</td>
<td>2.235</td>
<td>0.794</td>
<td>1.418</td>
<td>1.207</td>
</tr>
<tr>
<td>12</td>
<td>1.101</td>
<td>0.861</td>
<td><strong>0.719</strong></td>
<td>1.021</td>
<td>2.299</td>
<td>0.795</td>
<td>1.456</td>
<td>1.229</td>
</tr>
</tbody>
</table>

Note: \( PPC(h) \) Bold numbers indicate the lowest PPC for each horizon. RW is the random-walk model and AR is the first-order autoregressive model. In the RW and AR models, the time series are assumed to be mutually independent.
### Table 6: $PPC(\tau)$ for the Credit Spreads

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>DNS-CS(0)</th>
<th>DNS-CS(1)</th>
<th>DNS-CS(2)</th>
<th>CS(0)</th>
<th>CS(1)</th>
<th>CS(2)</th>
<th>RW</th>
<th>AR</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>1.131</td>
<td>0.714</td>
<td><strong>0.561</strong></td>
<td>1.049</td>
<td>2.618</td>
<td>0.668</td>
<td>1.291</td>
<td>1.167</td>
</tr>
<tr>
<td>36</td>
<td>1.052</td>
<td>0.677</td>
<td><strong>0.561</strong></td>
<td>0.986</td>
<td>2.227</td>
<td>0.624</td>
<td>1.128</td>
<td>1.019</td>
</tr>
<tr>
<td>60</td>
<td>0.937</td>
<td>0.801</td>
<td><strong>0.705</strong></td>
<td>0.990</td>
<td>1.691</td>
<td>0.739</td>
<td>1.188</td>
<td>1.038</td>
</tr>
<tr>
<td>84</td>
<td>0.907</td>
<td>0.862</td>
<td><strong>0.785</strong></td>
<td>0.962</td>
<td>1.443</td>
<td>0.804</td>
<td>1.072</td>
<td>0.964</td>
</tr>
<tr>
<td>120</td>
<td>0.734</td>
<td>0.608</td>
<td>0.571</td>
<td>0.741</td>
<td>1.253</td>
<td><strong>0.538</strong></td>
<td>0.895</td>
<td>0.788</td>
</tr>
</tbody>
</table>

Note: Bold numbers indicate the lowest PPC for each maturity. RW is the random-walk model and AR is the first-order autoregressive model. In the RW and AR models, the time series are assumed to be mutually independent.

### Figure 3: Posterior Predictive Density of the Yield Curve from the DNS-CS(2) Model

(a) One-month ahead  
(b) Three-month ahead  
(c) Six-month ahead  
(d) Twelve-month ahead

Note: The figure displays the 1, 3, 6, and 12 month-ahead posterior predictive density of the government yield, respectively.
To graphically demonstrate predicted yields and credit spreads, Figure 3 and Figure 4 plot the predictive densities for the yield curve and credit spread curve generated from the DNS-CS(2) model which is a model with the best forecasting performance in our study. As an example we choose the period February 2012 through January 2013 as our 12-month ahead out-of-sample period. Figure 3 shows that the median of predictive density is reasonably close to the actual yield curve and the 95% confidence bound include the actual yield curve for all forecast horizons. One can see that the government bond yields at short maturities are binding at the zero lower bound. Thus, our imposed zero lower bound restriction helps avoid the predictive distributions for short term bond yields with negative values. Figure 4 demonstrates that for this particular

\textit{Figure 4} Posterior Predictive Density of the Credit Spread Curve from the DNS-CS(2) Model

Note: The figure displays the 1, 3, 6, and 12 month-ahead posterior predictive density of the credit spread, respectively.
out-of-sample period the joint model has better predictions of the credit spread curve at 6 and 12-month horizons relative to shorter horizons. The figure also shows that the imposed zero lower bound restriction is not binding for the credit spread.

Overall, the credit spread curve information, accounting for structural changes in the factor process, and imposing non-zero restriction on the interest rate are all essential for yield curve forecast improvements.

2. Parameter Estimates

In this section, we report parameter estimates and further analyze causes of the structural breaks in the factor process and the sources of the prediction improvement in the DNS-CS(2) model. Figure 5 displays the posterior probability of the regimes over time. The model estimation clearly identifies two structural breaks in the joint dynamics of the yield curve and credit spread curve (i.e., probabilities of regimes are estimated at 0 or 1). The first break occurred in March 2008, and the second break occurred in January 2010. The first break is associated with the beginning of the period with near to zero short interest rate and the sharp increase in the credit spreads. In response to the financial crisis the demand for safe assets substantially increased and default and liquidity risk elevated, causing the credit spread level to rise. The second break is closely related to the default risk reduction as the U.S. economy begins to recover from the crisis. Thus, the change-points appear to be closely related to the business cycle. Therefore macroeconomic fundamentals such as output gap and inflation rate might contain additional information that can help predict the shape of the future yield and credit spread curves. We leave this question for the future research.

The above described changes in the dynamics of the yield curve and the credit spread curve are captured by changes in the estimates of the factor parameters across the regimes, reported in Table 7. These parameter estimates imply that the unconditional mean of the yield curve level factor, computed as $\frac{\mu_{\eta}}{1 - G_{\eta}}$, is much smaller in regime 1 than those in regimes 2 and 3. Meanwhile, the unconditional
Note: The figure displays the time series of probabilities of the three regimes, the time series of yields on 1-, 24-, and 120-month government bonds and the time series of 24-month and 120-month credit spreads.
**Forecasting the Term Structure of Government Bond Yields Using Credit Spreads and Structural Breaks**

**<Table 7> Intercept, AR Coefficients, and Transition Probabilities**

<table>
<thead>
<tr>
<th>Regime</th>
<th>Mean</th>
<th>s.e.</th>
<th>Ineff.</th>
<th>Mean</th>
<th>s.e.</th>
<th>Ineff.</th>
<th>Mean</th>
<th>s.e.</th>
<th>Ineff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 1</td>
<td>0.121</td>
<td>0.084</td>
<td>3.423</td>
<td>1.068</td>
<td>0.442</td>
<td>38.013</td>
<td>0.305</td>
<td>0.054</td>
<td>0.545</td>
</tr>
<tr>
<td>Regime 2</td>
<td>-0.012</td>
<td>0.033</td>
<td>0.766</td>
<td>-0.253</td>
<td>0.244</td>
<td>12.247</td>
<td>-0.286</td>
<td>0.057</td>
<td>0.560</td>
</tr>
<tr>
<td>Regime 3</td>
<td>-0.074</td>
<td>0.050</td>
<td>1.032</td>
<td>-0.443</td>
<td>0.543</td>
<td>5.892</td>
<td>-0.216</td>
<td>0.046</td>
<td>0.880</td>
</tr>
</tbody>
</table>

\( \mu_q \)

\( \gamma_q \)

\( p_{q,q'} \)

**Note:** In the state equation, \( \mu_q \) is the constant term and \( \gamma_q \) is the AR coefficient term. \( p_{q,q'} \) means the transition probabilities among regimes.

**<Figure 6> Factors**

(a) Factors in the government bond yields

(b) Factors in the credit spreads

**Note:** This figure plots the time series of the three factors of the government bond yields and the time series of the two factors in the credit spreads.
mean of the level factor of the credit spread curve is much higher during the crisis period than before and after the crisis.

The structural changes also capture changes in the dynamics of the slopes of the yield curve and the credit spread curve. <Figure 6> displays the time series of the model factors. The credit spread slope factor demonstrates that the credit spread curve tends to be downward sloping during recession periods in 2001 and 2008-2009 (i.e., the slope factor increased, or in other words, the slope decreased). In contrast, the yield curve tends to be steeper during recessions, when the Fed reduces the short-term interest rate.

Tables 7 - 10 report the estimation results for the model parameters. <Table 8> indicates that the factor volatilities are distinct across regimes. In particular, consistent with the yield and credit spread curve dynamics, the factors are most volatile in regime 2.

<Table 9> reports the estimation results of the correlations among the factor shocks in the three regimes. Considerable differences in the correlation estimates across regimes confirm instability of instantaneous interactions between the yield curve and credit spread curve. In the first regime, the shocks to the credit spread level was negatively correlated with the shock to the slope factor of the yield curve. In this regime, the U.S. economic growth was relatively strong and therefore the yield curve was strongly upward slopped and the credit spread was relatively tight. In the first regime, the shocks to the level factors of the yield curve and credit spread curve (\( f_t^L \) and \( x_t^L \)) were not strongly correlated, but after the first structural break the shocks to these factors became strongly negatively correlated. The strong negative correlation between the level shocks in the second period can be explained by the fight-to-quality when the demand for risk-free bonds increased while credit spreads dramatically increased. In the last regime, the correlation between shocks to the slope factor of the yield curve \( f_t^S \) and the level factor of the credit spread curve \( x_t^L \) was strongly positive, in contrast to the negative correction in the first regime. In this period, the credit risk decreased in response to the beginning of the economic recovery and the long-term risk-free rate decreased in response to the Fed's policy to keep the long-term interest rate at the low level to stimulate the economic growth.
### Table 8: Factor Volatility

<table>
<thead>
<tr>
<th></th>
<th>regime 1</th>
<th></th>
<th>regime 2</th>
<th></th>
<th>regime 3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>s.e.</td>
<td>ineff.</td>
<td>mean</td>
<td>s.e.</td>
<td>ineff.</td>
<td>mean</td>
</tr>
<tr>
<td>$f_{t}^L$</td>
<td>0.192</td>
<td>0.015</td>
<td>4.082</td>
<td></td>
<td></td>
<td>0.460</td>
</tr>
<tr>
<td>$f_{t}^S$</td>
<td>0.322</td>
<td>0.021</td>
<td>1.036</td>
<td></td>
<td></td>
<td>0.392</td>
</tr>
<tr>
<td>$f_{t}^C$</td>
<td>0.526</td>
<td>0.042</td>
<td>3.487</td>
<td></td>
<td></td>
<td>0.979</td>
</tr>
<tr>
<td>$X_{t}^L$</td>
<td>0.175</td>
<td>0.019</td>
<td>25.548</td>
<td></td>
<td></td>
<td>0.902</td>
</tr>
<tr>
<td>$X_{t}^S$</td>
<td>0.563</td>
<td>0.053</td>
<td>5.540</td>
<td></td>
<td></td>
<td>2.062</td>
</tr>
</tbody>
</table>

Note: $f_{t}^L$, $f_{t}^S$, $f_{t}^C$ indicate the time-varying level, slope, and curvature factors of government yield curve, respectively. $X_{t}^L$, $X_{t}^S$ indicate the credit spread level and slope factors, respectively.

### Table 9: Conditional Correlations

<table>
<thead>
<tr>
<th></th>
<th>$f_{t}^L$</th>
<th>$f_{t}^S$</th>
<th>$f_{t}^C$</th>
<th>$X_{t}^L$</th>
<th>$X_{t}^S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{t}^L$</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{t}^S$</td>
<td>-0.660</td>
<td>(0.056)</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{t}^C$</td>
<td>0.533</td>
<td>(0.089)</td>
<td>-0.273</td>
<td>(0.097)</td>
<td>1.000</td>
</tr>
<tr>
<td>$X_{t}^L$</td>
<td>-0.087</td>
<td>(0.117)</td>
<td>-0.214</td>
<td>(0.107)</td>
<td>-0.182</td>
</tr>
<tr>
<td>$X_{t}^S$</td>
<td>-0.327</td>
<td>(0.109)</td>
<td>0.159</td>
<td>(0.107)</td>
<td>-0.445</td>
</tr>
</tbody>
</table>

(a) regime 1

<table>
<thead>
<tr>
<th></th>
<th>$f_{t}^L$</th>
<th>$f_{t}^S$</th>
<th>$f_{t}^C$</th>
<th>$X_{t}^L$</th>
<th>$X_{t}^S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{t}^L$</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{t}^S$</td>
<td>-0.811</td>
<td>(0.111)</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{t}^C$</td>
<td>-0.590</td>
<td>(0.170)</td>
<td>0.615</td>
<td>(0.182)</td>
<td>1.000</td>
</tr>
<tr>
<td>$X_{t}^L$</td>
<td>-0.603</td>
<td>(0.160)</td>
<td>0.200</td>
<td>(0.245)</td>
<td>0.063</td>
</tr>
<tr>
<td>$X_{t}^S$</td>
<td>0.166</td>
<td>(0.246)</td>
<td>-0.374</td>
<td>(0.222)</td>
<td>-0.382</td>
</tr>
</tbody>
</table>

(b) regime 2

<table>
<thead>
<tr>
<th></th>
<th>$f_{t}^L$</th>
<th>$f_{t}^S$</th>
<th>$f_{t}^C$</th>
<th>$X_{t}^L$</th>
<th>$X_{t}^S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{t}^L$</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{t}^S$</td>
<td>-0.991</td>
<td>(0.004)</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_{t}^C$</td>
<td>-0.914</td>
<td>(0.032)</td>
<td>0.909</td>
<td>(0.033)</td>
<td>1.000</td>
</tr>
<tr>
<td>$X_{t}^L$</td>
<td>-0.702</td>
<td>(0.090)</td>
<td>0.679</td>
<td>(0.096)</td>
<td>0.874</td>
</tr>
<tr>
<td>$X_{t}^S$</td>
<td>-0.457</td>
<td>(0.201)</td>
<td>0.430</td>
<td>(0.207)</td>
<td>0.300</td>
</tr>
</tbody>
</table>

(c) regime 3

Note: $f_{t}^L$, $f_{t}^S$, $f_{t}^C$ are the time-varying level, slope, and curvature factors of government yield curve, respectively. $X_{t}^L$, $X_{t}^S$ are the credit spread level and slope factors, respectively.
<Table 10> Measurement Error Variances

<table>
<thead>
<tr>
<th></th>
<th>$\Sigma_1$</th>
<th>$\Sigma_2$</th>
<th>$\Sigma_3$</th>
<th>$\Sigma_4$</th>
<th>$\Sigma_5$</th>
<th>$\Sigma_6$</th>
<th>$\Sigma_7$</th>
<th>$\Sigma_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.0240</td>
<td>0.0015</td>
<td>0.0051</td>
<td>0.0028</td>
<td>0.0031</td>
<td>0.0059</td>
<td>0.0021</td>
<td>0.0189</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.0027</td>
<td>0.0003</td>
<td>0.0006</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0008</td>
<td>0.0004</td>
<td>0.0022</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\Sigma_9$</th>
<th>$\Sigma_{10}$</th>
<th>$\Sigma_{11}$</th>
<th>$\Sigma_{12}$</th>
<th>$\Sigma_{13}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.0917</td>
<td>0.0136</td>
<td>0.0453</td>
<td>0.1151</td>
<td>0.0081</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.0121</td>
<td>0.0033</td>
<td>0.0051</td>
<td>0.0119</td>
<td>0.0021</td>
</tr>
<tr>
<td>ineff.</td>
<td>8.0253</td>
<td>15.8237</td>
<td>2.5576</td>
<td>2.7895</td>
<td>20.3521</td>
</tr>
</tbody>
</table>

Note: The number of measurement errors denotes 13 maturities of government bond yields and credit spreads.

Overall, changes in all factor parameters across regimes play important roles in causing the structural breaks in the joint dynamics of the yield curve and the credit spread curve. In addition, differences in parameter estimates across regimes suggests that all regimes are distinct and non-repeating over time. To confirm it, in our preliminary analysis we conducted out-of-sample forecasting using regime-switching models with two regimes. We find that the regime-switching models produce yields forecasts that are worse than the change-point model. ³)

VI. Conclusion

We investigate whether credit spread curve information helps forecast the government bond yield curve. For this study we extend the yield curve modeling approach proposed by Diebold and Li (2006) to model the joint dynamics of the default-free bond yields and the credit spreads. Specifically, we model interactions between the government bond yields and the credit spreads by allowing

³) For conciseness of the paper we do not report the results from the regime-switching models. These results are available upon request.
instantaneous shocks to the yield curve factors and the credit spread curve factors to be correlated. We find that our proposed joint model produces substantially better forecasts of the risk-free yields relative to the standard three-factor DNS yield curve model. Our forecasting results suggest that the information contained in the credit spread curve helps improve the yield curve prediction at all considered forecast horizons.

In addition, our joint model allows for structural breaks in the factor process and imposes a zero lower bound restriction. We find that the model with two change-points in the feedbacks between the yield curve factors and the credit spread curve factors has the best out-of-sample forecasting performance among other considered model specifications. We estimate the change-points occurred in March 2008 and January 2010 and they appear to be associated with the period of the recent U.S. financial crisis. Also, our results show that a zero lower bound restriction helps avoid predictions of short-term bond yields with negative values in the near-to-zero short-term interest rate environment.
References


본 연구의 목적은 두 가지이다. 첫째는 만기별 신용스프레드에 내재된 정보가 미국 국채 수익률곡선 예측에 도움이 되는지를 분석하는 것이며, 두 번째는 미국 국채 시장과 회사채 시장간 동태적 상호관계에 구조적 변화를 고려하는 것이 국채 수익률 예측력 향상에 추가적인 이점이 있는지를 살펴보는 것이다.

 이를 위해, 본 연구는 동태적 넬슨-시겔(Nelson-Siegel) 모형을 확장한 국채 수익률곡선과 신용스프레드곡선 결합예측모형을 제시하고 효율적인 베이지안 MCMC 기법을 고안하였다. 본 연구의 주요성은 동태적 넬슨-시겔 모형에 비해 본 연구에서 제시한 결합예측모형이 보다 정확한 예측결과를 도출하였다. 더불어, 결합예측모형의 표본외 예측력은 두 채권시장의 구조변화를 고려함으로써 크게 향상되었다.

 본 연구에서 제시된 결합예측모형 및 추정기법은 제로금리제약과 반영하여 주로 미국 국채수익률 뿐만 아니라 일본 및 유로지역 금리 예측에도 실질적으로 적용될 수 있을 것이다.

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