Safe Assets

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Safe Assets

A safe asset is one whose real value is insulated from shocks, including declines in GDP associated with rare macroeconomic disasters. However, in a Lucas-tree world, the aggregate risk is given by the specified process for GDP and cannot be altered by the creation of safe assets. Therefore, in the equilibrium of a representative-agent version of this economy, the quantity of safe assets will effectively be nil. With heterogeneity in coefficients of relative risk aversion, safe assets may take the form of private bond issues from low-risk-aversion agents to high-risk-aversion agents. I work out the quantity of safe assets, the risk-free interest rate, and the equity premium in a model with Epstein-Zin-Weil preferences, heterogeneous coefficients of relative risk aversion, and log utility (intertemporal elasticity of substitution equal to one). In one example, safe assets are around 140% of annual GDP and 6% of economy-wide assets (comprising the capitalized value of the full GDP), the risk-free rate is 1.0% per year, and the equity premium is 4.2%. In the baseline model, Ricardian equivalence holds. Added (safe) government bonds have no effect on the economy's net quantity of safe assets or the risk-free rate and crowd out private bonds with a coefficient around -0.5.

In a Lucas-tree world (Lucas [1978]), the aggregate risk derives from the uncertainty in the process for GDP, which corresponds to the fruit that drops from the tree. This process may include rare macroeconomic disasters, which correspond to sharp and possibly permanent drops in the productivity or number of the trees. A safe asset in this world can be viewed as one whose real value is insulated from shocks, including the declines in GDP due to the rare disasters. However, if the GDP process is given, safe assets cannot mitigate overall risk but can only redistribute this risk across agents. In a representative-agent setting, the redistribution of aggregate risk cannot occur, and the economy's quantity of safe assets will effectively be nil.

To put this result another way, it is possible to construct safe assets by issuing risk-free private bonds, by creating a financial structure with risk-free tranches, by entering into a variety of insurance contracts, and so on. However, the creation of any of these safe assets always goes along with a corresponding expansion in the riskiness of (levered) claims on the underlying asset, which is the Lucas tree. In equilibrium, the representative agent ends up holding the representative share of the overall risk, and this overall magnitude is unaffected by the various financial arrangements. The bottom line is that a meaningful analysis of safe assets requires heterogeneity across agents.

Differences in the degree of risk aversion are a natural form of heterogeneity for a study of safe assets. The present analysis relies on the simplest possibility, where there are two types of agents; one with comparatively low risk aversion and another with comparatively high risk aversion. Specifically, when each agent i has a constant coefficient of relative risk aversion, γ_i, I assume $0 < \gamma_1 < \gamma_2$, so that agent 1 is more willing than agent 2 to absorb risk.

I focus on a model in which the desire to redistribute risk across agents is the central source of safe private assets. In equilibrium, the agent with relatively low risk aversion, agent 1, will issue safe bonds (or equivalent claims) that are held by the agent with relatively high risk aversion, agent 2. Correspondingly, agent 1 will own a disproportionate share of risky assets, which are equity claims on the Lucas tree. The quantity of safe assets in this economy equals the magnitude of the bonds issued by agent 1 and held by agent 2. The equilibrium amount of these assets depends on differences in risk aversion across the agents, levels of risk-aversion coefficients, and the characteristics of the stochastic process (including rare disasters) that generates aggregate GDP.

The equilibrium requires an enforcement mechanism for repayments of safe claims; that is, agent 1 has to make payments of principal and interest to agent 2 even in bad states of the world, such as realizations of macroeconomic disasters. Repayment mechanisms may involve collateral, liquidity, and contractual features related to the legal system. However, these mechanisms are not the subject of the present analysis, which focuses on the underlying supply of and demand for safe private assets. Related work that emphasizes aspects of liquidity, collateral, and asymmetric information includes Holmstrom and Tirole (1998) and Gorton and Ordoñez (2013). These issues could be brought into an extension of the present model.

A pure claim on the Lucas tree corresponds to unlevered equity. A match with the empirically observed high equity premium requires the expected rate of return on this equity to be substantially higher than the risk-free rate, which equals the rate of return on non-contingent, private bonds. Previous analyses with rare-disaster models, summarized in Barro and Ursúa (2012), have found that the replication of this high equity premium requires two conditions: first, a coefficient of relative risk aversion in the range of 3-4 or more (for a representative agent) and, second, the presence of fat-tailed uncertainty, such as a non-negligible potential for drops in GDP in the short run by more than 10%. The present analysis incorporates both of these features.

With the familiar specification where utility is time-separable and takes a power form, a coefficient of relative risk aversion of 3-4 or more implies an intertemporal elasticity of substitution (IES) of 1/3-1/4 or less, which seems unrealistically low. Specifically, the high value of γ_i needed to generate a realistic equity premium precludes the case of log utility in the sense of IES=1, which has advantages in terms of tractability. More generally, with the standard utility formulation, it is impossible for all agents to have log utility (IES=1) when coefficients of relative risk aversion differ across agents.

As is well known, the Epstein-Zin-Weil form of recursive utility, based on Epstein and Zin (1989) and Weil (1990), allows for a separation between the coefficient of relative risk aversion and the IES. Typically, this benefit from EZW comes at the cost of analytical complexity, when compared with the case of time-separable power utility. However, with heterogeneity in risk-aversion coefficients, the EZW form actually allows for a simpler analysis. The key property of EZW is that it admits high values of the γ_i coefficients that can differ across agents, while nevertheless having values of the IES that are of reasonable magnitude and the same for each agent. More specifically, the EZW case admits the possibility of IES=1 for each agent, thereby gaining great simplifications from log utility. (The rate of time preference, ρ, is also assumed to be the same for each agent.)

Previous models of asset pricing with two types of agents distinguished by their coefficients of relative risk aversion include Dumas (1989), Wang (1996), Chan and Kogan (2002), and Longstaff and Wang (2012). These analyses deal with time-separable, power utility, augmented in Chan and Kogan (2002) to include an external habit in household utility. In Wang (1996), one agent has log utility and the other has square-root utility—coefficients of relative risk aversion of 1 and ½, respectively. In Longstaff and Wang (2012), one agent has log utility and the other has squared utility—coefficients of relative risk aversion of 1 and 2, respectively. Gennaioli, Shleifer, and Vishny (2012) assume that one agent is risk neutral and the other has infinite risk aversion, and Caballero and Farhi (2014) use an analogous setup.

Section I works out a baseline model that derives equilibrium holdings of equity claims and private bonds by the two types of agents, distinguished by their degrees of risk aversion. The analysis emphasizes a tractable case with Epstein-Zin utility, where coefficients of relative risk aversion are high but utility is logarithmic in the sense that the intertemporal elasticity of substitution is one. Section II carries out quantitative analyses based on specifications of the underlying parameters, including coefficients of relative risk aversion and the characteristics of the macro-disaster process. The focus is on how the equilibrium quantity of safe assets relates to these underlying parameters. As an example, when agent 1 has $\gamma_1 = 3.5$ and agent 2 has $\gamma_2 = 5.5$, safe assets are around 140% of GDP and 6% of economy-wide assets (comprising the capitalized value of all of GDP), the risk-free rate is

1.0% per year, and the equity premium is 4.2%.

Section III relates the gross quantity of private bonds to the net quantity corresponding to loans from agent 2 to agent 1. Section IV introduces public debt. An important point is that added government bonds create more safe assets while simultaneously creating a corresponding amount of "safe liabilities" in the form of the present value of taxes. In the baseline setting, Ricardian equivalence holds, in the sense that changes in the quantity of government bonds do not affect the economy's net quantity of safe assets or the risk-free rate. More specifically, the model predicts that an increase in government bonds by 1 unit crowds out private bonds by around 0.5 units. If the government is superior to the private sector in its ability to commit to pay principal and interest on bonds and in using the tax system to collect from debtors, Ricardian equivalence may fail.

Ⅰ. Baseline Model

The model is set up for convenience in discrete time. For some purposes, the period length can be viewed as a year. However, in constructing formulas for equilibrium rates of return, it is appropriate to think of the period length as approaching zero.

Agent i, for i=1, 2, has an EZW utility function, given by:

$$(1-\gamma_i)\cdot U_{it} = \left\{ C_{it}^{1-\theta} + \left(\frac{1}{1+\rho}\right)[(1-\gamma_i)E_t U_{i,t+1}]^{(1-\theta)/(1-\gamma_i)} \right\}^{(1-\gamma_i)/(1-\theta)} \quad (1)$$

The coefficients of relative risk aversion satisfy $0 < \gamma_1 \leq \gamma_2$. The IES, $1/\theta>0$, and the rate of time preference, $\rho > 0$, are the same for the two agents. I simplify by having $\theta=1$ in the main analysis. This condition corresponds to the usual case of log utility and makes the price of equity independent of parameters that describe expected growth and uncertainty.[1)] Another result with log utility is that the consumption of each agent, C_1 and C_2, is given approximately by ρ times a measure of each agent's assets.

Parts of the basic structure parallel Longstaff and Wang (2012). The single Lucas tree generates real GDP of Y_t in year t. This GDP is consumed by the two agents:

$$C_{1t} + C_{2t} = Y_t \tag{2}$$

Ownership of the tree is given by K_{1t} and K_{2t}, which must add to full ownership, normalized to one:

$$K_{1t} + K_{2t} = 1 \tag{3}$$

I use a convention whereby K_{it} applies at the end of period t, after the payment of the dividend, $K_{i,t-1} \cdot Y_t$, to agent i in period t. (This timing convention will not matter as the length of the period becomes negligible.) The price of the tree in period t in units of consumables is P_t.

The stochastic process that generates Y_t corresponds to previous rare-disaster analyses, except for the omission of a normally-distributed business-cycle shock, which is quantitatively unimportant. The probability of disaster is the constant p per year. With probability 1-p, real GDP grows over one year by the factor 1+g, where g>0 is constant. With probability p, a disaster occurs and real GDP grows over one year by the factor (1+g)·(1-b), where b>0 is the size of a disaster. In the present simplified setting, disasters last for only one "period" and have a single size.

The analysis can be readily extended to allow for a time-invariant size distribution of disasters, as discussed in Barro and Ursúa (2012). With more complexity, the analysis can be modified to allow disasters to have stochastic duration and cumulative size and to be followed by a tendency for recovery in the sense of above-normal growth rates (Nakamura, Steinsson, Barro, and Ursúa

1) This result means that the expected rate of return on equity, r^e, is independent of uncertainty parameters. Therefore, with $\theta=1$, all of the effects of uncertainty parameters on the equity premium work through the risk-free rate, r^f, rather than r^e. We know from previous analysis of this i.i.d. setting with a representative agent that the equity premium is independent of the parameter θ. Therefore, in this context, the setting of $\theta=1$ would not affect the model's implications for the equity premium.

[2013]). Other extensions that are feasible include time variation in the disaster probability, p (as in Gabaix [2012]), and the growth-rate parameter, g (as in Bansal and Yaron [2004]).

In the present setting, I assume p=0.04 per year, which corresponds to the empirical frequency of disasters (represented by declines in per capita GDP over short periods by at least 10%). The effective disaster size—in the sense of the single value that generates an equity premium corresponding roughly to the full size distribution of disasters—is around b=0.32. The growth-rate parameter, intended to correspond to real per capita GDP or consumption, is set at g=0.025 per year.

I focus on two forms of asset claims. Aside from equity claims, K_{it}, on the tree, I consider a non-contingent, one-period private bond, B_{it}. The quantity B_{it} is negative for a borrower (issuer of a bond) and positive for a lender (holder of a bond). Since I consider a closed economy, the total quantity of these private bonds, when added up across the two types of agents, is always zero:

$$B_{1t} + B_{2t} = 0 \tag{4}$$

The model assumes a perfect private credit market, in the sense of ignoring possibilities of default and neglecting any transaction costs associated with interest and principal payments; that is, with collecting on loans.[2] In this case, bonds pay off at the risk-free interest rate for period t, denoted r_t^f. The amount of principal and interest received or paid on bonds by agent i in period t is $(1+r_t^f)\cdot B_{i,t-1}$.

The menu of assets and financial contracts could be extended to include structured finance, stock options, macro-disaster insurance, etc., and to have risk-free bonds of varying maturities. However, the setup with equity and one-period risk-free private bonds is sufficient to characterize the equilibrium in the present model.[3]

Each agent's budget constraint for period t is:

2) In the cases considered, debtors always have sufficient total assets to make the prescribed principal and interest payments on bonds.

3) This result also holds in Wang (1996, p. 80) and Longstaff and Wang (2012, p. 3175).

$$C_{it} + P_t K_{it} + \mathrm{B_{it}} = (Y_t + P_t) K_{i,t-1} + (1 + r_t^f)\, \mathrm{B_{i,t\text{-}1}} \tag{5}$$

The choice for period t of C_{it} and the asset allocation (K_{it}, B_{it}) occur when Y_t, P_t, and r_{t+1}^f are known but Y_{t+1} and P_{t+1} are unknown.

Let R_t represent the gross return on any asset (equity, risk-free bonds) between periods t and t+1. This return equals $(Y_{t+1}+P_{t+1})/P_t$ for equity and $(1+r_{t+1}^f)$ for bonds. Each agent seeks to maximize expected utility, given in equation (1), subject to the budget constraint in equation (5) and the levels of initial assets. The first-order optimization conditions for each agent and each type of asset can be expressed by means of a perturbation argument for periods t and t+1 as:

$$(E_t U_{i,t+1})^{\left(\frac{\theta-\gamma_i}{1-\gamma_i}\right)} = \left(\frac{1}{1+\rho}\right) E_t\left[U_{i,t+1}^{\left(\frac{\theta-\gamma_i}{1-\gamma_i}\right)} \cdot \left(\frac{C_{i,t+1}}{C_{it}}\right)^{-\theta} \cdot R_{t+1}\right] \tag{6}$$

This expression simplifies in straightforward ways under log utility, θ=1.

Previous analyses (Giovannini and Weil [1989], Obstfeld [1994]), Barro [2009]) found in a representative-agent model with i.i.d. shocks (as in the present setting) that the realized utility, U_{t+1}, can be expressed as a positive constant multiplying $(C_{t+1})^{(1-\gamma)}/(1-\gamma)$. This result suggests looking for an approximate solution to the present two-agent model in which $U_{i,t+1}$ is a positive constant (possibly different for each agent) multiplying the analogous object for agent i, $\frac{(C_{i,t+1})^{1-\gamma_i}}{1-\gamma_i}$. When this condition holds, equation (6) can be rewritten as:

$$\left[E_t\left(\frac{C_{i,t+1}}{C_{it}}\right)^{1-\gamma_i}\right]^{\left(\frac{\theta-\gamma_i}{1-\gamma_i}\right)} = \left(\frac{1}{1+\rho}\right)\cdot E_t\left[\left(\frac{C_{i,t+1}}{C_{it}}\right)^{-\gamma_i} \cdot R_{t+1}\right] \tag{7}$$

When R_{t+1} equals the risk-free return, $1+r_{t+1}^f$, and θ=1, equation (7) implies

$$1+r_{t+1}^f = (1+\rho)\cdot \frac{E_t\left(\frac{C_{i,t+1}}{C_{it}}\right)^{(1-\gamma_i)}}{E_t\left(\frac{C_{i,t+1}}{C_{it}}\right)^{-\gamma_i}} \tag{8}$$

Thus, a key implication of the first-order conditions is that, in equilibrium, the right-hand side of equation (8) has to be the same for each agent; that is, for γ_1 and γ_2, respectively. In other words, the prospective paths of uncertain consumption levels for the two agents have to accord with the differences in the coefficients of relative risk aversion.

I now use the agents' budget constraints to get expressions for the terms involving consumption growth in equation (8). Under log utility, $\theta=1$, each agent's consumption in period t is approximately the multiple ρ of that agent's resources for period t:[4]

$$C_{it} \approx \rho \cdot [(Y_t + P_t) \cdot K_{i,t-1} + (1+r_t^f)\, B_{i,t-1}] \tag{9}$$

Using equation (9), along with the budget constraint in equation (5), leads eventually to the condition:[5]

$$\left(\frac{C_{i,t+1}}{C_{it}}\right)^{-\gamma_i} \approx \left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma_i} \cdot \left[1 + \frac{\gamma_i \rho B_{it}}{Y_t K_{i,t-1}}\right] - \left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma_i - 1} \cdot \left[\frac{\gamma_i \rho B_{it}}{Y_t K_{i,t-1}}\right] \tag{10}$$

Analogous conditions hold for the terms $\left(\frac{C_{i,t+1}}{C_{it}}\right)^{1-\gamma_i}$, with $1-\gamma_i$ replacing $-\gamma_i$ on the right-hand side of equation (10).

Recall that the stochastic process that generates Y_t involves systematic growth at rate g with a Poisson probability p for (permanent) disasters of size b. This process implies that the expectations of the terms involving Y_{t+1}/Y_t in equation (10) are given by:

4) See Giovannini and Weil (1989, Appendix B). Adding up equation (9) for the two agents and using the conditions from equations (3) and (4) that equity holdings add to 1 and bond holdings add to zero leads to the result $P_t=Y_t \cdot (1-\rho)/\rho$. As the length of the period becomes negligible, this condition approaches $P_t=Y_t/\rho$.

5) The main assumption underlying the approximation in equation (10) is that the ratio of the magnitudes of risk-free bonds, $B_{i,t-1}$ and B_{it}, to the value of equity holdings, $K_{i,t-1} \cdot (Y_t/\rho)$, is much less that one for each agent. [This approximation is not so good in some cases. Although the present results should be illustrative, the numerical analysis should be redone to gain greater precision.]

$$E_t\left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma_i} = (1+g)^{-\gamma_i}\cdot[1-p+p\cdot(1-b)^{-\gamma_i}] \quad (11)$$

$$E_t\left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma_i-1} = (1+g)^{-\gamma_i-1}\cdot[1-p+p\cdot(1-b)^{-\gamma_i-1}] \quad (12)$$

Ⅱ. Quantitative Analysis

I start the analysis in period 0 with asset holdings of $B_{10}=B_{20}=0$ and $K_{10}=K_{20}=0.5$. That is, there is initially no private bond issue, and equity holdings are evenly distributed across the two types of agents. Heuristically, since $\gamma_1 < \gamma_2$, there would be an incentive in this initial position for agent 1 to issue risk-free bonds, $B_{11}<0$, in period 1, which end up being held by agent 2, so that $B_{21}>0$. That is, agent 1 would borrow from agent 2 on a safe basis. Correspondingly, agent 1 would use its bond issue to increase its share of equity, $K_{11}>0.5$, which is effectively bought from agent 2, so that $K_{21}<0.5$. In a richer model, this process of safe credit creation would be important for the determination of the equilibrium amount of aggregate investment.

The adjustments of bond and equity positions shift some risk from the high-risk-aversion agent 2 to the low-risk-aversion agent 1. However, the process does not entail complete risk shifting; rather, enough bond issue occurs so that the resulting stochastic paths of future consumption for each agent make the right-hand side of equation (8) the same for each agent i. The value for r^f_{t+1} at each date then follows from equation (8). Given a specification of parameters, especially γ_1 and γ_2, it is straightforward to use equation (8), along with equations (10)-(12), to determine numerically the dynamic path of B_{it}, K_{it}, and r^f_{t+1} that corresponds to a set of realizations for Y_t.

Aside from the coefficients of relative risk aversion, γ_1 and γ_2, the baseline parameter values, listed in the notes to Table 1, are $\rho=0.04$ per year (rate of time preference), $g=0.025$ per year (growth-rate parameter), $p=0.04$ per year (disaster probability), and $b=0.32$ (effective disaster size). These values accord with the prior empirical analysis summarized in Barro and Ursúa (2012). These parameter value imply that the expected growth rate is

$$g^* = g - p \cdot b = 0.0122 \text{ per year} \tag{13}$$

I also assume log utility, $\theta=1$.

1. A Representative Agent

Table 1 considers cases with a representative agent, where $\gamma_1 = \gamma_2 = \gamma$. In these cases, B_{it} and K_{it} stay constant over time at their initial values ($B_{10}=B_{20}=0$ and $K_{10}=K_{20}=0.5$), regardless of the realizations of Y_t. Because of log utility, the expected rate of return on equity, r^e, is fixed, independently of γ, at $\rho+g^*$, where $\rho=0.04$ per year and g^* is given in equation (13). The resulting value of r^e is 0.0522 per year. A higher γ lowers the risk-free rate, r^f, and, thereby, raises the equity premium. Specifically, r^f ranges from 0.038 at $\gamma=2$ to -0.035 at $\gamma=6$. An unlevered equity premium between 0.03 and 0.06 (corresponding roughly to historical data) requires γ to be between 3.5 and 5. For a given γ, r^f is fixed over time, regardless of the realizations of Y_t. This risk-free rate is a shadow rate in the sense that no borrowing and lending occur in equilibrium. That is, no safe assets are created in this representative-agent environment.

2. Heterogeneity in Risk Aversion

Table 2 allows for differences between γ_1 and γ_2. The assumption in this table, which provides a baseline for the analysis, is $\gamma_1=3.5$ and $\gamma_2=5.5$. For illustrative purposes, the process for Y_t assumes no disasters between years 1 and 5, so that Y_t grows at $g=0.025$ per year. The level of Y_t rises accordingly from a normalized value of 1 in year 1 to 1.10 in year 5. A disaster shock of size $b=0.32$ is assumed to arrive in year 6, so that Y_t falls from 1.10 to 0.77. Then Y_t grows again at $g=0.025$ per year, reaching a level of 0.85 in year 10.

The magnitude of bonds, $|B_{1t}|$, jumps to 156% of GDP and 6.2% of economy-wide assets in year 1. Note that economy-wide assets equal P_t because equity holdings always add to one, $K_{1t}+K_{2t}=1$, and bond holdings always add to zero, $B_{1t}+B_{2t}=0$. The equity price is proportional to GDP: $P_t=y_t/\rho$. Therefore, with $\rho=0.04$ per year, the ratio of $|B_{1t}|$ to annual GDP is always 25 times the ratio to economy-wide assets. Note also that total assets correspond to the capitalization of the entire flow of GDP, effectively including the value of all human capital as well as physical capital. For this reason, ratios of safe assets to economy-wide assets tend to look small when compared to ratios expressed relative to annual GDP.

Table 2 shows that, in the absence of a disaster, $|B_{1t}|$ reaches a little over 140% of GDP and about 5.7% of economy-wide assets in year 5. Correspondingly, the fraction of equity, K_{1t}, held by the low-risk-aversion agent is around 57%, above the starting share of 50%. This agent consumes a little more than half of GDP (and total consumption); that is, C_{1t}/Y_t in year 5 equals 51.4%. This ratio equals the share of economy-wide assets held by agent 1 (because, with log utility, each agent has the same propensity to consume, ρ, out of assets). The reason that this share exceeds 0.5 is that the low-risk-aversion agent's average rate of return on assets exceeds that of the high-risk-aversion agent. That is, agent 1 holds more than his share of equity, financed by safe debt, and r^e exceeds r^f. This comparison in rates of return across agents applies from year 1 to year 5 in the absence of a disaster, but it applies on average even with the realization of disasters (at frequency p with size b).

The high-risk-aversion agent, agent 2, holds around 43% of the value of equity claims in year 5. This share may seem high, but it arises because equity comprises the capitalized value of all of GDP, including human capital. Therefore, the result can be interpreted as agent 2 holding most or all of the ownership rights in his own human capital, rather than selling these rights (to the extent permissible by law!) to agent 1.

The risk-free rate, r^f, in Table 2 for year 5 is a little over 1% per year. This rate compares with $r^e=5.2\%$ and, therefore, to an (unlevered) equity premium around 4.2%. Note that the value for r^f around 1% in Table 2, when $\gamma_1=3.5$ and $\gamma_2=5.5$, is below the value of 1.9% in Table 1, corresponding to $\gamma_1=3.5$ and $\gamma_2=3.5$. That is, the value $\gamma_2=5.5$ means that, with respect to the determination of r^f and the equity premium, the economy operates like a representative-agent setup with a coefficient of relative risk aversion higher than γ_1; specifically, with a value of $\gamma_1=\gamma_2\approx 4$ (see Table 1).

In Table 2, the disaster shock of size 0.32 in year 6 sharply lowers the level of Y_t (from 1.10 to 0.77). The effects on the equilibrium in year 6 and subsequent years derive not from the drop in Y_t, per se, but rather from the change in relative asset holdings across the agents. Specifically, C_{1t}/Y_t—which equals the share of economy-wide assets held by the low-risk-aversion agent—falls from 0.514 in year 5 to 0.491 in year 6. This change occurs because agent 1 holds more than 50% of equity claims and is, therefore, particularly susceptible to a disaster-induced drop in dividends corresponding to the fall in GDP.

This channel—whereby a smaller share of assets is owned by agents with comparatively low risk aversion—provides a mechanism for persisting effects of macroeconomic disasters on the market equilibrium, including the magnitudes of safe assets and the levels of r^f.[6)] (In a richer model, there would also be persisting effects on investment.) This persistence applies even though the underlying shocks to GDP are i.i.d. In the present case, however, the magnitudes of these persisting effects turn out not to be large. For example, in Table 2, the ratio of the magnitude of agent 1's bonds to GDP goes from 1.422 in year 5 to 1.413 in year 6, then up to 1.466 in year 7, and then down to 1.444 in year 10. The risk-free rate rises from 0.0102 in year 5 to 0.0103 in year 6 but then falls to 0.0096 in year 7 before returning to 0.0100 in year 10.

The model as it stands is not quite stationary because, even when allowing for occasional disasters, the expected return on assets for agent 1 exceeds that for agent 2. Thus, there is a tendency for K_1 and C_1/Y to rise slowly over time in Table 2. Stationarity could be achieved by having agents of each type die off randomly, with replacement by new agents with a random (50-50) choice of coefficient of relative risk aversion of either γ_1 or γ_2. The replacement agents could inherit the assets of their predecessors, who could be viewed as parents. However, a full analysis would require choices of consumption and asset holdings to take account

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6) An analogous source of persisting influence on the equity premium arises in Chan and Kogan (2002). Longstaff and Wang (2012) study persisting influences on asset prices and the quantity of credit.

of the possibility of dying off with replacement by children whom one may or may not care about.

Another possibility is for no one literally to die off but for people randomly to experience moments when they consider a shift in coefficient of relative risk aversion to either γ_1 or γ_2. Each destination could have a 50-50 chance of being picked. This mechanism would not require dealing with inheritance and altruism but would necessitate having decisions on consumption and asset holdings depend on the potential for a shift in one's own attitude toward risk.

3. Alternative Combinations of Coefficients of Relative Risk Aversion

Table 3 shows the effects from once-and-for-all differences in the coefficients of relative risk aversion, γ_1 and γ_2. Case I, γ_1=3.5 and γ_2=5.5, is the baseline specification used in Table 2. The results shown in Table 3 are for year 5, after the effects from the starting position ($B_{10}=B_{20}=0$ and $K_{10}=K_{20}=0.5$) have had time to die off. Cases II (γ_1=4, γ_2=6) and III (γ_1=5, γ_2=7) consider γ_i above the baseline values, while maintaining the gap of 2.0 between the two values. Cases IV (γ_1=3, γ_2=5) and V (γ_1=2, γ_2=4) are analogous for lower γ_i.

Higher values of the γ_i coefficients associate with a lower risk-free rate, r^f, and a correspondingly higher equity premium (because r^e is fixed when $\theta=1$). Similarly, lower values of the γ_i match up with a higher risk-free rate, r^f, and a correspondingly lower equity premium. Given the spread by 2.0 between γ_1 and γ_2, values of γ_1 much above 4 or much below 3 conflict with an observed (unlevered) equity premium between 0.03 and 0.06.

Lower values of the γ_i coefficients, such as case IV in Table 3, correspond to more risk shifting; that is, the magnitude of agent 1's bonds is 157% of GDP and 6.3% of economy-wide assets in case IV, compared to 142% and 5.7% in the baseline (case I). Similarly, K_1 is 0.583 in case IV, compared to 0.573 in the baseline.

Higher values of the γ_i coefficients, such as case II, associate with less risk shifting. The magnitude of agent 1's bonds is then 130% of GDP and 5.2% of economy-wide assets, and K_1 is 0.566.

One conclusion is that the results are unrealistic if γ_1 is low, say 2, even if γ_2 is substantial, such as 4. The implied equity premium is then less than 2% per year, inconsistent with the data. The model's predictions would be even more inaccurate if γ_1 were closer to zero (risk neutrality). That is, satisfactory results with regard to asset returns cannot be generated by having a set of nearly risk-neutral agents paired up with risk-averse agents.

Cases VI and VII set γ_1 at its baseline value, 3.5, but consider greater or smaller spreads between γ_2 and γ_1. When γ_2 is also 3.5, we know from Table 1 that $B_1=0$, $K_1=0.5$, and $r^f=0.0188$. When $\gamma_2=4$, as in case VI, the results move part way from the baseline toward this setting with equal γ's. That is, the magnitude of agent 1's bonds is 52% of GDP and 2.1% of economy-wide assets, compared to 142% and 5.7% in the baseline. The share of economy-wide equity held by agent 1, K_1, is 0.526, compared to 0.573 in the baseline. The risk-free rate, r^f, in case VI is 0.0154, compared to 0.0102 in the baseline (case I) and 0.0188 when $\gamma_2=3.5$ (Table 1).

When $\gamma_2=7$ is paired with $\gamma_1=3.5$ in case VII, the magnitude of agent 1's bonds reaches 172% of GDP and 6.9% of economy-wide assets. The share of economy-wide equity held by agent 1, K_1, is 0.590, and the risk-free rate, r^f, is 0.0088.

Case VIII returns to the baseline specification of $\gamma_1=3.5$ and $\gamma_2=5.5$, but considers a once-and-for-all lower disaster probability; that is, p=0.02, rather than 0.04. (A smaller disaster size, b, would have a similar effect.) The risk-free rate, r^f, is substantially higher, 0.0372, and the equity premium correspondingly smaller. Substantially more risk shifting occurs than in the baseline setting: in case VIII, the magnitude of agent 1's bonds is 160% of GDP and 6.4% of economy-wide assets, and K_1 is 0.585.

Ⅲ. Gross versus Net Lending

The bond holdings, B_1, shown in Tables 2 and 3 correspond to net safe lending from the high-risk-aversion agent, agent 2 (who holds bonds of $B_2=-B_1$), to the low-risk-aversion agent, agent 1. There is a sense, however, in which gross bond issuance is not pinned down, because the model would allow for unlimited borrowing and lending within groups. That is, agent 1 could effectively issue an arbitrary amount of bonds to himself, and analogously for agent 2.

If the model includes an infinitesimal amount of transaction costs for bond issuance and collection of interest and principal, then this borrowing and lending within groups would not occur in equilibrium in the present model. In this case, the quantity of bond issuance, $|B_1|$, shown in Tables 2 and 3 would be the unique equilibrium for the gross amount outstanding.

If transaction costs for collecting on loans (bonds) are substantial, the quantity of net bond issuance and the risk-free rate would differ significantly from the values found in Tables 2 and 3. Moreover, the risk-free rate received by lenders (group 2) would deviate from that paid by borrowers (group 1). For example, if transaction costs were prohibitive, the results would correspond to autonomy for groups 1 and 2 and correspond, thereby, to the results shown in Table 1. The quantity of net bond issuance would be zero, and the quantity of capital held by each group would be 0.5. When γ_1=3.5, the shadow risk-free rate for group 1 would be 0.0188, whereas if γ_2=5.5, the shadow risk-free rate for group 2 would be -0.0220. That is, members of group 1 would be willing to pay a rate of return of 0.0188 per year at the margin on risk-free borrowing, whereas members of group 2 would be willing to accept a rate of return of -0.0220 per year at the margin on risk-free lending.[7] However, no net issue of safe debt occurs because of the prohibitive transaction costs.

In practice, it would be difficult to measure empirically the economy's quantity of safe assets in the sense of the counterpart to the magnitude of net lending from group 2 to group 1. One issue is that borrowing and lending may occur within groups in a richer model. In this case, the observed gross amount of private bonds would overstate the quantity of safe assets. Another consideration is that an array of financial arrangements—including structured finance, stock options, and insurance contracts—can mimic safe bonds. In this respect, the observed amount of non-contingent private bonds would likely understate the quantity of safe assets. In any event, measuring safe assets empirically in a conceptually meaningful way is a major challenge.[8]

7) Note that, in the present model, there is nothing special about a risk-free rate of zero.

8) Gorton, Lewellen, and Metrick (2012) provide empirical measures of safe assets, but their criteria for including assets in this category seem arbitrary.

Ⅳ. Government Bonds and Ricardian Equivalence

Suppose that the government issues one-period bonds with characteristics corresponding to those of private bonds. The interest rate on government bonds held from year t to year t+1 must then be r^f_{t+1}, the same as that on private bonds. The simplest approach is for the government to make a distribution in the form of lump-sum transfers of bonds in year t in the aggregate quantity B^g_t. This distribution is assumed to go 50-50 to members of groups 1 and 2. The aggregate principal and interest, $(1+r^f_{t+1})B^g_t$, is paid out to government bondholders in period t+1. This payout is financed by lump-sum taxes, levied equally in period t+1 on members of groups 1 and 2.

What is the impact of this government bond issue on private bond issue, the risk-free interest rate, and so on? The government bond issue does not affect the households' first-order conditions involving the risk-free rate, r^f_{t+1}, which appear in equation (8). There is also no effect on households' budget constraints in equation (5) (updated to apply to periods t and t+1), once one factors in the transfer payments in year t and the taxes levied in year t+1. Therefore, it is immediate that the equilibrium involves the same net borrowing and lending as before between agents 1 and 2, the same risk-free interest rate, r^f_{t+1}, the same equity price, $P_t=Y_t/\rho$, and the same expected rate of return on equity, $r^e=\rho+g^*$. That is, the equilibrium features Ricardian equivalence with respect to (net) quantities of safe assets and the various rates of return.

There are multiple possibilities with respect to gross debt that support the equilibrium. One possibility is that groups 1 and 2 each hold 50% of the government bonds issued in year t. These holdings correspond to the present value of the (certain) tax liabilities imposed on each group. That is, government bonds and tax liabilities cancel with respect to creating safe assets; no safe assets are created on net. The quantity of net private borrowing and lending, corresponding to $|B_{1t}|$, is then the same as before, and the same risk-free interest rate supports this equilibrium.

Alternatively, all of the government bonds could be held, in equilibrium, by group 2. Members of group 2 then have additional safe assets (in the form of government bonds) that exceed the present value of their added tax liabilities (which equal 50% of the government bonds). Correspondingly, members of group 1 hold no government bonds but have a present value of tax liabilities also equal to 50% of the bonds. Therefore, group 1 has additional tax liabilities (50% of the government bonds) that exceed their added safe assets in the form of government bonds (which are nil in this case). This result works if the net private borrowing by group 1 from group 2 falls by an amount equal to 50% of the government bonds. In this case, the net position of group 1 with respect to group 2 is the same as before. The only difference is that some of the borrowing and lending between the groups is purely private, while some works through the government as intermediary (collecting taxes from group 1 and using the proceeds to pay principal and interest on half of the government bonds held by group 2).

As in the previous section, the indeterminacies with respect to gross debt are resolved if there is an infinitesimal amount of transaction costs for bond issuance and collection of interest and principal. These transaction costs are assumed at this point to be the same for private and public bonds. In this case, the unique equilibrium will be the one just described whereby group 2 holds all of the added government bonds, and the issue of government bonds crowds out 50% of the private bonds that would otherwise have existed. (This result assumes that the gross quantity of private bonds outstanding was initially greater than 50% of the added government bonds.) In any event, the main result is that Ricardian equivalence holds, in the sense that public debt issue does not alter the quantity of safe assets in the relevant net sense and also does not affect equilibrium rates of return.

A surprising conclusion from the model is that the crowding-out coefficient for private bonds with respect to public bonds is -0.5, not -1.0. This result came from a model with a number of simplifying assumptions; notably, there were just two groups characterized by their coefficients of relative risk aversion, γ_i, and the incidence of the present value of taxes net of transfers associated with the government bond issue was the same for each group. However, the predicted crowding-out coefficient around 0.5 does not depend on these assumptions holding precisely. For example, the restriction to two groups is unimportant.[9)] The main assumption that seems to matter is that there is little correlation across groups between γ_i and the share of taxes net of transfers that applies to group i.

The predicted crowding-out effect relates to the analysis of Krishnamurthy and Vissing-Jorgensen (2013, p. 1), who advanced the proposition "that government debt … should crowd out the net supply of privately issued short-term debt." Their theory implied crowding out but did not pin down a specific coefficient. They tested the crowding-out hypothesis on U.S. data for 1914-2011 and found (Table 4, Panel A) that an increase in the quantity of net U.S. government debt had a significantly negative effect on the net short-term debt created by the private financial sector. Remarkably, their estimated coefficient was very close to -0.5, the value predicted by the model described above.

The results are different and Ricardian equivalence may not hold exactly if the government is superior to the private sector in the technology of creating safe assets.[10] In particular, the government might be able to commit better than private agents to honoring payments of principal and interest on its bonds and can also use the coercive power of the tax system to ensure the financing of these payments. On the other hand, a private lending arrangement requires only that group 1 makes principal and interest payments to group 2 in period t+1, whereas the public setup entails the government raising taxes from group 1 in period t+1 and then using the proceeds to pay off group 2. Once the distorting influences from taxation are considered, it is not obvious that the public process entails lower

9) Suppose, for example, that there are four groups of agents, where $\gamma_1<\gamma_2<\gamma_3<\gamma_4$. Suppose further that the initial equilibrium involves private bond holdings of B_1=-100, B_2=-50, B_3=50, B_4=100. Assume that the government issues 4 units of bonds, with the present value of taxes rising by 1 unit for each group. In this case, the two private borrowers go, in equilibrium, to B_1=-99 and B_2=-49, thereby preserving their positions for bond holdings net of tax liabilities (of 1 each) at -100 and -50, respectively. The two private lenders go, in equilibrium, to overall bond positions (inclusive of government bonds) of B_3=51 and B_4=101, thereby preserving their positions for bond holdings net of tax liabilities at 50 and 100, respectively. Note that the additional 4 units of government bonds crowd out the total of private bonds by 2 units.

10) Caballero and Farhi (2014, p.3) make this assumption, although they do not clarify the specific elements that underlie the government's superior technology: "Public debt … plays a central role … as typically the government owns a disproportionate share of the capacity to create safe assets while the private sector owns too many risky assets. … The key concept then is that of fiscal capacity: How much public debt can the government credibly pledge to honor should a major macroeconomic shock take place in the future?" They also do not consider that public debt issue creates additional "safe liabilities" in the form of taxes that match the added safe assets in a present-value sense.

"transaction costs" overall.[11)]

Another consideration is that the expansion of public debt and associated taxation is poorly targeted in the sense that 50% of the added government bonds—held by group 2—match the added present value of tax liabilities for group 2 and, therefore, do not serve to shift risk toward group 1. Only the remaining 50% of government bonds corresponds to this shifting of risk. In contrast, all of the private bonds issued by group 1 and held by group 2 associate with risk shifting.

V. Gold and other Commodities as Safe Assets

Gold and other precious durable commodities, such as silver and platinum, are often viewed as forms of comparatively safe assets. However, as noted in Barro and Misra (2013), real returns on gold are as volatile as stocks in the period since 1975, although real gold returns have a covariance with growth rates of GDP and consumption that may be negative and is surely much smaller than that for stock returns.

In the model, the underlying economy-wide risk corresponds to the uncertainty in the process for GDP, Y_t, which corresponds to the fruit from the Lucas tree. Any application of this model to macroeconomic data would require Y_t to be identified with a composite of a variety of goods and services. In this sense, the services from gold or other commodities would constitute forms of goods and services that are already factored into the aggregate variable Y_t. Then, depending on how the values of these commodities covary with the rest of GDP, the variable Y_t inclusive of the commodity service flows might be more or less volatile than the variable gauged exclusively of these flows. In any event, the analysis would be the same as that already carried out. In particular, the determinants of net safe borrowing and lending and the risk-free rate would follow the form of the analysis in Tables 2 and 3.

11) Even if the observed interest rate on government bonds is lower than that on private bonds, the overall transaction costs—including the distorting effects from taxation—associated with the public process might exceed that for the private process.

Ⅵ. Extensions of the Model

An important extension is to achieve stationarity; specifically, to avoid the conclusion that the low-risk-aversion agent 1 holds all of the wealth asymptotically. A natural approach for making the model stationary is to have agents "die off" randomly and then be replaced with a fifty-fifty choice of a new agent with coefficient of relative risk aversion equal to γ_1 or γ_2. In this setting, people may or may not be linked altruistically to members of the next generation. An alternative metaphor has people living forever but occasionally reaching states at which they shift their coefficients of relative risk aversion randomly between γ_1 and γ_2.

In a stationary version of the model, it is straightforward to carry out comparative-statics exercises concerning the impact of changes in the various parameters, including γ_1 and γ_2, the disaster probability, p, and the disaster size, b. Another important parameter is the rate at which incumbent agents disappear and replacement agents arrive. These various parameters can be related to the equilibrium (steady-state) quantity of safe assets, expressed relative to GDP or to total assets. These predictions can then be compared with empirical observations on quantities of safe assets, such as those generated by Gorton, Lewellen, and Metrick (2012).

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<Appendix>

Table-1. Representative-Agent Economy
(Single Coefficient of Relative Risk Aversion)

$\gamma_1 = \gamma_2 = \gamma$	r^f	r^e
1	0.0462	0.0522
1.5	0.0422	0.0522
2	0.0373	0.0522
2.5	0.0314	0.0522
3	0.0243	0.0522
3.5	0.0156	0.0522
4	0.0051	0.0522
4.5	−0.0076	0.0522
5	−0.0230	0.0522
5.5	−0.0418	0.0522
6	−0.0645	0.0522

Notes: When the coefficients of relative risk aversion are the same, $\gamma_1=\gamma_2=\gamma$, the equilibrium quantities of bonds, B_1 and B_2, are zero and the ownership of equity is evenly distributed, $K_1=K_2=0.5$. The table shows the equilibrium risk-free rate, r^f, for each value of γ. The calculations assume that the growth-rate parameter is g=0.025 per year, the rate of time preference is ρ=0.04 per year, the disaster probability is p=0.04 per year (corresponding in the historical data to contractions of per capita GDP by at least 10%), and the effective disaster size is b=0.32. The expected growth rate is $g^*=g-p\cdot b=$ 0.0122 per year. The reciprocal of the IES is θ=1. The expected rate of return on equity, given θ=1, is $r^e=\rho+g^*=0.0522$ per year, which is independent of γ. The price of equity is $P=Y/\rho=25\cdot Y$.

In this representative-agent case, the risk-free rate can be written in closed form, if $\gamma\neq 1$, as:

$$r^f = \rho + \theta g + p\left(\frac{\theta-1}{\gamma-1}\right) - p(1-b)^{-\gamma} + p\left(\frac{\gamma-\theta}{\gamma-1}\right)(1-b)^{1-\gamma}.$$

If $\theta=1$, as γ approaches 1, r^f approaches $\rho+g-pb/(1-b)$.

Table-2. Dynamics of Safe Assets, Equity Ownership, Consumption, and Risk-Free Rate (Baseline case with γ_1=3.5 and γ_2=5.5)

Year	Y	$\mid B_1 \mid /Y$	$\mid B_1 \mid$ /assets	K_1	C_1/Y	r^f	r^e
1	1	1.555	0.0622	0.565	0.500	0.0082	0.0522
2	1.025	1.433	0.0573	0.564	0.504	0.0101	0.0522
3	1.0506	1.435	0.0574	0.567	0.508	0.0101	0.0522
4	1.0769	1.429	0.0572	0.570	0.510	0.0102	0.0522
5	1.1038	1.422	0.0569	0.573	0.514	0.0102	0.0522
Disaster shock of size 0.32							
6	0.7694	1.413	0.0565	0.549	0.491	0.0103	0.0522
7	0.7886	1.466	0.0584	0.554	0.493	0.0096	0.0522
8	0.8083	1.456	0.0582	0.557	0.496	0.0098	0.0522
9	0.8285	1.450	0.0580	0.560	0.500	0.0099	0.0522
10	0.8492	1.444	0.0577	0.563	0.503	0.0100	0.0522

Notes: Real GDP, Y, is set to 1 in year 1, then grows at 0.025 per year until a disaster hits in year 6. The disaster shock in year 6 reduces Y by the fraction 0.32, after which Y grows at 0.025 per year through year 10. The coefficients of relative risk aversion for the two agents are γ_1=3.5 and γ_2=5.5. The other parameter values, including θ=1, are indicated in the notes to Table 1. Safe assets in each year satisfy $B_1+B_2=0$; equity claims on the tree in each year satisfy $K_1+K_2=1$. The economy starts in period 0 with $B_1=0$ and $K_1=0.5$. As in Table 1, the expected rate of return on equity is fixed at $r^e=\rho+g^*=0.0522$ per year. The price of equity is $P_t = Y_t / \rho = 25 \cdot Y_t$. The table shows for each year the magnitude of bonds (debt) expressed as a ratio to GDP or economy-wide assets (which equals P_t). Also shown are the amount of agentís 1ís equity ownership, K_1, which is a fraction of the fixed total of 1; the ratio of agent 1ís consumption to GDP (which equals total consumption); and the risk-free rate, r^f. Note that, with log utility (θ=1), the ratio C_1/Y equals the share of economy-wide assets held by agent 1.

Table-3. Safe Assets, Equity Ownership, Consumption, and Risk-Free Rate: Effects from Different Combinations of Coefficients of Relative Risk Aversion

$\mid B_1 \mid /Y$	$\mid B_1 \mid$ /assets	K_1	C_1/y	r^f	r^e
I. $\gamma_1 = 3.5$, $\gamma_2 = 5.5$					
1.422	0.0569	0.573	0.514	0.0102	0.0522
II. $\gamma_1 = 4$, $\gamma_2 = 6$					
1.300	0.0520	0.566	0.512	−0.0001	0.0522
III. $\gamma_1 = 5$, $\gamma_2 = 7$					
1.107	0.0440	0.539	0.514	−0.0242	0.0522
IV. $\gamma_1 = 3$, $\gamma_2 = 5$					
1.569	0.0628	0.583	0.516	0.0194	0.0522
V. $\gamma_1 = 2$, $\gamma_2 = 4$					
1.996	0.0798	0.606	0.520	0.0344	0.0522
VI. $\gamma_1 = 3.5$, $\gamma_2 = 4$					
0.515	0.0206	0.526	0.505	0.0154	0.0522
VII. $\gamma_1 = 3.5$, $\gamma_2 = 7$					
1.719	0.0688	0.590	0.518	0.0088	0.0522
VIII. $\gamma_1 = 3.5$, $\gamma_2 = 5.5$, $p = 0.02$					
1.605	0.0642	0.585	0.516	0.0372	0.0586

Notes: The values shown correspond to year 5 in Table 2, when real GDP is Y=1.1038. Case I uses the baseline parameter values from Tables 1 and 2, including γ_1=3.5, γ_2=5.5, and θ=1. Cases II-VII use alternative values for γ_1 and γ_2, as shown. Case VIII returns to γ_1=3.5 and γ_2=5.5 but assumes that the disaster probability is p=0.02, rather than p=0.04.

<Abstract in Korean>

Robert J. Barro

안전자산은 실질가치가 충격(드문 거시경제적 재난에 따른 GDP의 감소 등)에 영향을 받지 않는 자산을 말한다. 루카스(Lucas)의 트리(tree) 모형에서 경제전체의 위험은 나무에서 산출되는 열매 양의 불확실성을 의미하므로, 안전자산은 경제전체의 위험을 재배분 할 수는 있으나 경제전체의 위험 자체를 줄일 수는 없다. 특히 모든 경제주체가 동일하다면 경제전체의 위험은 재배분 될 수 없고 이에 따라 안전자산의 규모는 0이 될 것이다. 만약 상대적 위험회피 성향이 이질적인 두 종류의 경제주체가 존재한다면, 안전자산은 민간 채권의 형태로 위험회피성향이 낮은 경제주체가 이를 매각하고 위험회피성향이 높은 경제주체가 이를 매입하게 될 것이다. 본 논문은 Epstein-Zin-Weil 선호, 이질적인 상대적 위험회피 성향, 시점간 대체탄력성(intertemporal elasticity of substitution)이 1이 되는 로그 효용함수를 가정한 뒤, 경제 내 안전자산의 규모, 무위험이자율(risk-free interest rate), 그리고 주식프리미엄(equity premium)이 어떻게 결정되는지 보이는 것을 목적으로 한다. 일례로 기본가정 하에서 모형은 안전자산의 규모가 GDP의 140%, 경제전체 총자산의 6%가 되며, 무위험이자율은 1.0%, 그리고 주식프리미엄은 4.2%가 될 수 있음을 보인다. 기본가정 하에서 리카도의 대등정리(Ricardian Equivalence)가 성립한다. 즉 정부가 안전자산을 공급하더라도 경제 내 안전자산의 순규모와 무위험이자율에는 영향을 주지 못한다. 다만 정부가 1의 안전자산을 공급할 경우 민간채권을 0.5만큼 감소하는 구축효과가 있다.