

# Loan Rate Differences across Financial Sectors: A Mechanism Design Approach

**Byoung-Ki Kim\*, Jun Gyu Min\*\***

The views expressed herein are those of the authors and do not necessarily reflect the official views of the Bank of Korea. When reporting or citing this paper, the authors' names should always be explicitly stated.

---

\* Senior Economist, Monetary Policy Department, The Bank of Korea, Tel: +82-2-759-4941, Email: [bkkim@bok.or.kr](mailto:bkkim@bok.or.kr).

\*\* Senior Economist, Economic Research Institute, The Bank of Korea, Tel: +82-2-759-5480, E-mail: [jgmin@bok.or.kr](mailto:jgmin@bok.or.kr).

The authors thank Byung Kwun Ahn, Myeonghwan Cho, Jae Joon Han, Joo Yong Jun, Jong Ku Kang, Jinill Kim, Dongyeol Lee, Manjong Lee, Joon Myung Woo, Hyun Chang Yi, Myung-Soo Yie and seminar participants at the Bank of Korea, at 2015 Korean Economic Association Joint Conference, at Financial-Industry Organization Study Group of Korea, and at Korea University for their useful comments and suggestions.

# Contents

|   |    |
|---|----|
| I . Introduction .....                        | 1  |
| II . Model .....                              | 4  |
| III . Co-existence of Financial Sectors ..... | 13 |
| IV . Conclusion .....                         | 25 |
| References .....                              | 27 |

# Loan Rate Differences across Financial Sectors: A Mechanism Design Approach

This paper shows that discrete and vastly different loan rates offered by different types of financial firms constitute, in fact, an elaborate mechanism that makes borrowers tell the truth regarding their ability to pay back loan principal and interest. Suppose that once a borrower fails to pay back a loan to a bank, he cannot borrow from any banks again and must contact higher-interest charging credit finance companies to get a new loan. This creates a well-defined incentive for borrowers: pay back and remain in the banks' loan market vs. do not pay back and move to, say, credit finance companies' loan market in which a higher loan rate is charged. This mechanism does not require the financial firms to verify even if the borrower declares bankruptcy, and therefore is more efficient than a standard debt contract à la Townsend (1979) in terms of verification cost. As the interest rates offered by different types of financial firms should be well aligned in order to prevent the deception of borrowers, we can also analyze how many different types of financial firms, that is, how many discrete and different loan rates, can co-exist in the economy.

**Keywords:** Debt contract, Mechanism, Loan rates, Co-existence of financial sectors

**JEL Classification:** D82, G21, G23

## I. Introduction

Generally, loan interest rates offered by various financial firms are vastly different across financial firm types.<sup>1)</sup> In Korea, for example, around the end of 2015 banks offered around 5%, special credit finance companies 15%, mutual savings banks 25% and consumer loan finance companies 30% for their loans. We explore the cross-sectional structure of loan rates across financial firm types. Why do they offer such different loan interest rates? Should the difference be wide or narrow? How many financial firm types that offer different loan interest rates can co-exist in the economy?

There are many factors, of course, that possibly affect loan interest rates: differences in risk, regulation, market structure, and terms and conditions of loans, etc. One is naturally tempted to argue that the loan rate differences mainly reflect differences in borrowers' credit ratings. But then one must scrutinize and explain the reason that credit ratings are different across borrowers, which is essentially the same as the original problem. From a theoretical point of view, this is arguably a kind of tautology unless credit rating is endogenously determined in the model.

With this kind of tautology in mind, we will address the above questions by studying a specific debt contract. In doing so, we assume that agents are identical in terms of ability. Specifically, each agent, seeking a fixed amount of loan each period, has an ex ante indistinguishable project with the same probabilistic outcome on the same support. An agent can close the loan contract depending on the outcome of the project. He can also intentionally lie to the lender that the project performed poorly, even if it did not. The previous sector in which he did not honor the contract remains publicly observable, but not the fact that he lied in breaching the contract. Agents differ only in their history of paying back the debt, and this history places the agents into different

---

1) In this paper, 'financial sectors' and 'financial firm types' are used interchangeably. As it will become clear later, financial firms are not distinguishable in terms of cost structure apart from the different loan rates they offer. Different loan rates create different financial sectors in which corresponding financial firm type, that is, those offering the corresponding loan rate, operate.

financial sectors. The specific financial sector in which an agent currently borrows can be defined as the credit rating for that agent.

There is a well-known truth-telling mechanism in a costly state verification model: a standard debt contract with lender's full verification, as in Townsend (1979). Verification takes place every time the borrower declares default (due to poor realization of ex ante profitable production plan, for example). Later, Mookherjee and Png (1989) show that random verification is more efficient from the lender's cost minimization perspective. All verifications must be random, and if the borrower is audited and verified to be truthful, the borrower must be rewarded so that all borrowers, including those who pay back the debt, strictly prefer to be audited.

We add another truth-telling mechanism in this paper. Specifically, we will show that the loan rate differences across financial firm types, in fact, constitute an elaborate truth-telling mechanism regarding the borrower's ability to pay back the principal and interest. Suppose that once a borrower fails to pay back the loan to a bank, he cannot borrow from any banks again, and must contact higher-interest-charging credit finance companies to get a new loan. This creates a well-defined incentive for borrowers: pay back and remain in the banks' loan market vs. don't pay back and move to the credit finance companies' loan market in which higher loan rates are charged. Contrary to a standard debt contract as in Townsend (1979), this mechanism does not require the financial firms to verify always when the borrower declares bankruptcy, and hence is more readily applicable to an environment in which verification is very costly or practically very difficult. Indeed, in less developed economies, the degree of loan rate differences is more prominent. Banerjee and Duflo (2011), among others, classify the extreme variability of loan rates within the same village or town as one of the stylized facts about credit markets in developing economies. Further, our model nicely nests Townsend's standard debt contract as a special case in terms of the number of co-existent financial sectors: when full verification takes place, only one financial sector can exist and two or more financial sectors cannot co-exist. Contrary to Mookherjee and Png (1989), who consider random verification with side payments in the form of rewards

between the lender and borrower, we do not allow side payments. Instead, we study a specific form of debt contract with exogenously given verification probability, and focus on what happens as the verification probability, which is posited as a given technology to financial firms in this paper, gradually decreases from one to zero in an economy consisting of finite lenders and a continuum of borrowers. The partial verification in this paper translates into the co-existence of different types of financial firms which offer discretely different loan rates in the economy. Further, the less probable the verification (i.e. the less developed and/or the more restrictive the verification technology), the smaller the number of co-existent financial sectors and the wider the gaps between loan rates offered across different financial sectors. By deriving these interesting results, this paper sheds some light on how we can better understand cross-sectional differences of loan rates.

This paper is related to the literature on dynamic debt contracts. Chang (1990) and Monnet and Quintin (2005) study properties of an optimal debt contract between one borrower and one lender to finance a project that lasts for multiple finite periods, with a focus on the structure of payment streams from the borrower to the lender. Chang (1990) shows, using a two-period model of costly state verification, that the optimal debt contract can be interpreted as a bond contract having a call (prepayment) option to pay up and close the contract at date one instead of going into date two. Monnet and Quintin (2005) use a finite period model to show a similar result that front-loading payments are always weakly optimal.<sup>2)</sup> This paper explores the dynamic environment from a different angle and focuses on explaining the co-existence of financial sectors that offer different loan rates, instead of deriving the optimal structure of debt contract between one borrower and one lender.

This paper is also related to credit rationing literature pioneered by Stiglitz and Weiss (1981) in the sense that not all of the agents with the same ability can borrow. A marked difference is that, in our model, credit is rationed in each and every financial sector. As a result, agents face different loan rates even though

---

2) Albuquerque and Hopenhayn (2004), and Cooley et al. (2004) also present a similar result based on limited contract enforceability.

their abilities are all identical, and in the end some agents cannot borrow from any financial firms. In other words, this paper derives a stationary non-symmetric allocation in which different types of financial firms offer different loan rates. Ghosh et al. (1999) cover credit rationing with multiple lenders in which a defaulting borrower can switch to a different lender. They focus on the analysis of the stationary symmetric equilibrium in which all lenders offer the same loan rate, paying attention to the probability that the new lender uncovers the past default of the borrower.

This paper is organized as follows. Section 2 describes the model, sets up the value functions of the agents and discusses incentive compatibility and individual rationality conditions. Section 3 derives an equilibrium in which the greatest number of financial firm types can co-exist. We will also cover the standard debt contract with full verification and will show that different types of financial firms cannot co-exist in the equilibrium once financial firms verify on every occasion when the borrower declares bankruptcy. Some simulation results and comparative statics analyses are also provided. Section 4 concludes.

## II. Model

Time is discrete and indefinite. Each period consists of two sub-periods, sub-period 0 and sub-period 1. The common discount rate between period  $t$  and period  $t+1$  is  $\beta$ . There is no discount between sub-periods within a period. There are  $N$  financial sectors  $S_1, S_2, \dots, S_N$ , each occupied by the corresponding financial firm type:  $B_1, B_2, \dots, B_N$ .<sup>3)</sup> We assume, without loss of generality,  $r_1 \leq r_2 \leq \dots \leq r_N$ , where  $r_j$  is the loan rate charged by financial firm  $B_j$ .<sup>4)</sup> Financial firms finance any amount of loans they make from outside the economy with zero interest rate at sub-period 0 in each period. They have to repay the funds they have raised at sub-period 1 in the same period.

---

3) For the moment, we posit that financial firm  $B_j$  is operating in financial sector  $S_j$ . Later we allow financial firm  $B_j$  to choose its own sector to operate in.

4) Alternatively, one can think of one big financial firm composed of subsidiaries. Each subsidiary, in this case, could be thought of representing a corresponding financial sector.

There is a continuum of risk neutral agents  $i \in [0, 1]$ . Let  $K_j$  denote the mass of agents in financial sector  $S_j$  so that  $\sum_j K_j = 1$ ,  $K_j \equiv \int_j 1 di$ . In each period, agent  $i$  borrows  $\alpha$  and invests in his project at sub-period 0. The project returns  $y_i \sim U[0, \bar{y}]$  at sub-period 1 in the same period. Agent  $i$  in sector  $S_j$ ,  $j \in \{1, 2, \dots, N\}$  reports  $\tilde{y}_i \in \{Yes, No\}$  to the financial firm  $B_j$ .<sup>5)</sup> If  $\tilde{y}_i = \text{'Yes'}$  then the debt (principal and interest) is cleared and agent  $i$  remains in sector  $S_j$ . If  $\tilde{y}_i = \text{'No'}$  then there are two cases. (i) An audit can take place, and all the ownership of the corresponding project is transferred from agent  $i$  to the financial firm. An audit by  $B_j$  can fully recover true  $y_i$  at a cost of  $c$ . Or, (ii) an audit does not take place, the financial firm cannot recover at all, and agent  $i$  keeps  $y_i$  in his pocket. In either case, from the next period on, agent  $i$  is forced to move down to sector  $S_{j+1}$  with probability  $m$ . If this happens, financial firms can observe the previous financial sector,  $S_j$ , in which agent  $i$  has failed to repay the debt, but they cannot observe whether agent  $i$  has told a lie in breaching the contract. Simply put, the former is publicly observable, but not the latter. For convenience, we assume that there is no capital, no collateral and no aggregate uncertainty. We do not allow rewards or side payments between financial firms and agents, but we do allow probabilistic verification (random auditing). In a stationary state, inflow into and outflow from any sector  $S_j$ ,  $j \in \{1, 2, \dots, N\}$  must be equal. We assume that inflow into sector  $S_1$  is equal to outflow from sector  $S_N$  and that agents kicked out of sector  $S_N$  must leave the economy. Note that the credit rating of an agent, which can be defined by the financial sector the agent currently borrows in, falls unidirectionally. In other words, an agent starts to borrow in sector  $S_1$  when he enters the economy and moves all the way down to sector  $S_N$  before he is forced to leave the economy.<sup>6)</sup>

5) Note that following the convention by letting  $\tilde{y}_i \in [0, 1]$  does not make any difference in the context of this paper.

6) This assumption keeps the model simple and tractable. Although it seems a bit strong, Christiano et al. (2014) make use of a set up similar in spirit. In particular, entrepreneurs in their model are deprived of all their wealth eventually since they are destined to experience failure with positive probability each period.



A government agency, central bank or supervisory agency, for example, constructs  $(r_1, r_2, \dots, r_N)$  that generates a ‘stationary allocation’ in which the incentive compatibility and individual rationality conditions of all the agents are satisfied while profits of all the financial firms are weakly positive and the agent population of each financial sector remains the same across the periods.<sup>7)</sup> In doing so, the government agency's objective is to design a financial system — that is, to construct  $(r_1, r_2, \dots, r_N)$  — composed of as many financial sectors as possible.<sup>8)</sup> Financial firms operate as long as profits are greater or equal to zero. In particular, profit maximization of financial firms and competition among them will not be seriously considered and zero-profit conditions for financial firms will not be imposed. Later, however, we will discuss an environment in which the government agency is taken out of the economy and financial firms are allowed to maximize their profits. Interestingly, as competition among profit-maximizing financial firms intensifies, it also starts to generate financial sectors across which discretely different loan rates are charged.

Now we turn to value functions of agents that formally describe incentive compatibility and individual rationality conditions. Let  $q \in [0, 1]$  denote the probability of verification when agents declare bankruptcy. We treat  $q$  as given and posit that it represents existing verification technology. Consider agent  $i$  in sector  $S_j$ ,  $j \in \{1, 2, \dots, N\}$  with a realized project return  $y_i$ . We can write

$$V_j(y_i) = \max \left\{ y_i - \alpha(1 + r_j) + \beta E[V_j(\cdot)], \right. \\ \left. q[\beta(1 - m)E[V_j(\cdot)] + \beta m E[V_{j+1}(\cdot)]] \right. \\ \left. + (1 - q)[y_i + \beta(1 - m)E[V_j(\cdot)] + \beta m E[V_{j+1}(\cdot)]] \right\}. \quad (1)$$

The first line of the above equation represents the agent's payoff when  $\tilde{y}_i = \text{‘Yes.’}$  Agent  $i$  enjoys the realized project return after paying back the debt and stays in the same financial sector next period. The second and third line

7) The government agency can impose this set of loan rates tightly, for example by giving each financial firm permission to operate in the corresponding financial sector.

8) Once we obtain the equilibrium with the maximum number of financial sectors, constructing an equilibrium with less number of financial sectors is easy, as becomes clear later.

depict the payoff arising when  $\tilde{y}_i = \text{'No,}'$  weighted by the verification probability. The second line shows that, when an audit takes place, all the return of agent  $i$ 's existing project is transferred to the financial firm  $B_j$ , and agent  $i$  moves down to the next financial sector  $S_{j+1}$  with probability  $m$  and remains in the current financial sector  $S_j$  with probability  $1 - m$ . The difference in the third line is that, when an audit does not take place, agent  $i$  can keep the current project return  $y_i$ . Of course, this value function is subject to the following incentive compatibility condition:

$$\begin{aligned} y_i - \alpha(1 + r_j) + \beta E[V_j(\cdot)] &\geq q[\beta(1 - m)E[V_j(\cdot)] + \beta m E[V_{j+1}(\cdot)]] \\ &\quad + (1 - q)[y_i + \beta(1 - m)E[V_j(\cdot)] + \beta m E[V_{j+1}(\cdot)]], \\ &\text{for } y_i \geq \alpha(1 + r_j). \end{aligned} \quad (2)$$

Focusing on the truth-telling mechanism as the revelation principle states, we can derive the expected value of  $V_j(\cdot)$ , in terms of the expected value of  $V_{j+1}(\cdot)$ .

$$\begin{aligned} E[V_j(\cdot)] &= \frac{1}{\bar{y}} \int_{\alpha(1+r_j)}^{\bar{y}} y_i - \alpha(1 + r_j) + \beta E[V_j(\cdot)] dy_i \\ &\quad + \frac{1}{\bar{y}} \int_0^{\alpha(1+r_j)} q[\beta(1 - m)E[V_j(\cdot)] + \beta m E[V_{j+1}(\cdot)]] \\ &\quad + (1 - q)[y_i + \beta(1 - m)E[V_j(\cdot)] + \beta m E[V_{j+1}(\cdot)]] dy_i \\ &= \frac{1}{\bar{y}} \int_{\alpha(1+r_j)}^{\bar{y}} y_i - \alpha(1 + r_j) + \beta E[V_j(\cdot)] dy_i \\ &\quad + \frac{1}{\bar{y}} \int_0^{\alpha(1+r_j)} (1 - q)y_i + \beta(1 - m)E[V_j(\cdot)] + \beta m E[V_{j+1}(\cdot)] dy_i. \end{aligned} \quad (3)$$

Manipulating and rearranging Equation (3) yield:

$$\begin{aligned} 2(1 - \beta)\bar{y}E[V_j(\cdot)] &= [\bar{y} - \alpha(1 + r_j)]^2 + (1 - q)\alpha^2(1 + r_j)^2 \\ &\quad - 2\beta m\{E[V_j(\cdot)] - E[V_{j+1}(\cdot)]\}\alpha(1 + r_j). \end{aligned} \quad (4)$$

Note that agent  $i$  with realization  $y_i < \alpha(1+r_j)$  in hand cannot repay the debt not because of the incentive compatibility but because of physical impossibility. In other words, the incentive compatibility condition, Equation (2), could hold with strong inequality for all realization  $y_i$  in the support. We are, however, particularly interested in the maximum number of financial sectors that can co-exist, and therefore we let the incentive compatibility condition, Equation (2), bind with equality at  $y_i = \alpha(1+r_j)$  for  $j \in \{1, 2, \dots, N\}$ . Then, from Equation (2), we have<sup>9)</sup>

$$\beta m \{E[V_j(\cdot)] - E[V_{j+1}(\cdot)]\} = (1-q)\alpha(1+r_j), \text{ for } j = \{1, 2, \dots, N\}. \quad (5)$$

The incentive compatibility condition for agents basically tells us that the decrease of value as agents move one financial sector down from sector  $S_j$  to sector  $S_{j+1}$  should be sufficiently large. As agents move further down the sectors, the decrease in value gets bigger because of the increase in the loan rate  $r_j$ . This incentive compatibility condition provides an explicit form of the expected value of remaining in financial sector  $S_j$ . Plug Equation (5) into Equation (4) to get the explicit form of the expected value.

$$E[V_j(\cdot)] = \frac{1}{2(1-\beta)\bar{y}} \{[\bar{y} - \alpha(1+r_j)]^2 - (1-q)\alpha^2(1+r_j)^2\},$$

$$\text{for } j = \{1, 2, \dots, N\}. \quad (6)$$

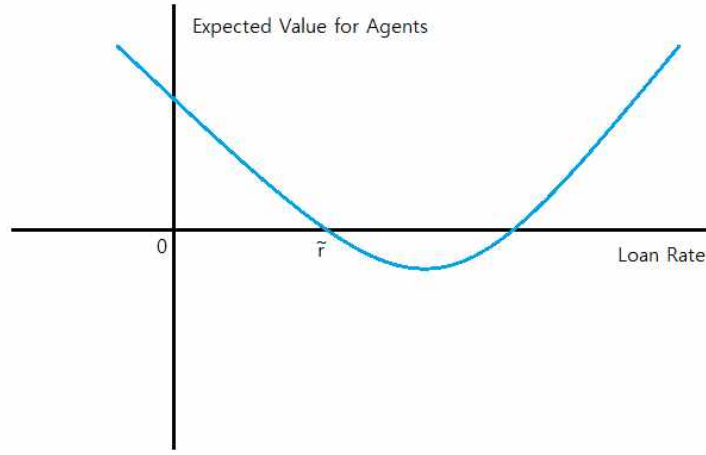
Let us now turn to the individual rationality condition for agents. This condition,  $E[V_1(\cdot)] \geq \dots \geq E[V_N(\cdot)] \geq 0$ , together with Equation (4) implies

$$E[V_N(\cdot)] = \frac{1}{2(1-\beta)\bar{y}} \{[\bar{y} - \alpha(1+r_N)]^2 - (1-q)\alpha^2(1+r_N)^2\} \geq 0. \quad (7)$$

---

9) Obviously,  $E[V_{N+1}(\cdot)] = 0$ .

Figure 1: Loan Rate and the Value for Agents



Note that Equations (6) and (7) indicate that the expected value is first decreasing and then increasing in the loan rate since the expected value is a quadratic equation. Obviously, we are only interested in the region of loan rate in which the expected value is positive and decreasing in the loan rate. It is easy to see that the expected value is positive when the loan rate is zero and that  $E[V(\cdot)] = 0$  has two distinct positive roots or one repeated positive root. Let  $\tilde{r}$  denote the smaller one of those two distinct roots or the repeated root itself. We have

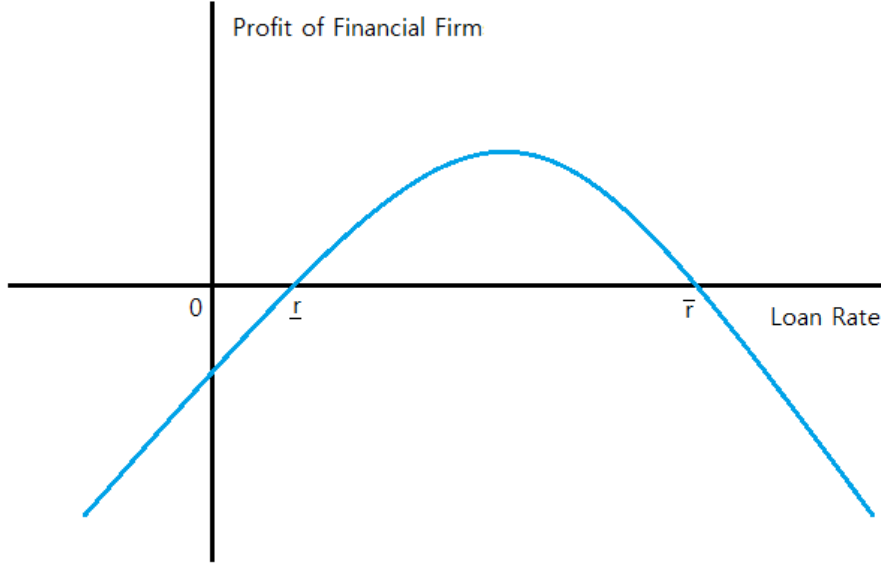
$$1 + \tilde{r} = \bar{y}(1 - \sqrt{1 - q}) / (q\alpha). \quad (8)$$

Then the region of loan rate in which the expected value is weakly positive,  $r \in [0, \tilde{r}]$ , is well-defined and not empty (Figure 1).

Consider the profit of financial firm  $B_j$  operating in sector  $S_j$ .

$$\begin{aligned} \Pi_j &= \frac{K_j}{y} \left\{ [\bar{y} - \alpha(1 + r_j)][\alpha(1 + r_j) - \alpha] + \int_0^{\alpha(1 + r_j)} q(y_i - \alpha - c) + (1 - q)(-\alpha) dy_i \right\} \\ &= \frac{\alpha K_j}{2y} \left\{ -\alpha(2 - q)(1 + r_j)^2 + 2(\bar{y} - qc)(1 + r_j) - 2\bar{y} \right\} \equiv K_j \pi_j. \end{aligned} \quad (9)$$

Figure 2: Loan Rate and the Profit of the Financial Firm



Note that the mass of agents with project return  $y_i$  sufficient enough to repay the debt is  $K_j[\bar{y} - \alpha(1 + r_j)]/\bar{y}$ . The first term inside the brace in the first line, hence, represents profit financial firm  $B_j$  can extract from these agents. For each of those with project return not sufficient to repay the debt, financial firm  $B_j$  recovers  $y_j$  at a cost  $c$  through an audit with probability  $q$  or recovers nothing if an audit does not take place with probability  $1 - q$ . In the former case the profit should be  $y_i - \alpha - c$ , and in the latter case  $-\alpha$ . Focus on terms inside braces and note that the condition to have two different real roots can be written as:

$$(\bar{y} - qc)^2 - 2\alpha(2 - q)\bar{y} > 0. \quad (10)$$

We make an assumption on parameter values so that the above inequality always holds.

**Assumption 1** We assume that  $\bar{y} > 4\alpha$  and  $\alpha > c$ .

Assumption 1 states that the maximum output that a project can achieve ( $\bar{y}$ ) should be sufficiently larger than the initial amount of a loan ( $\alpha$ ), and the

verification cost ( $c$ ) should be smaller than the loan ( $\alpha$ ). This condition is not strong and can be thought very plausible. Under Assumption 1, two distinct positive roots (denoted by  $\underline{r}$ ,  $\bar{r}$ ) that make Equation (9) equal to zero exist. In particular,

$$1 + \underline{r} = \left[ \bar{y} - qc - \sqrt{(\bar{y} - qc)^2 - 2\alpha\bar{y}(2 - q)} \right] / [\alpha(2 - q)], \quad (11)$$

$$1 + \bar{r} = \left[ \bar{y} - qc + \sqrt{(\bar{y} - qc)^2 - 2\alpha\bar{y}(2 - q)} \right] / [\alpha(2 - q)]. \quad (12)$$

For  $r \in [\underline{r}, \bar{r}]$ , the profit of the financial firm is weakly positive (Figure 2). In what follows, we assume that Assumption 1 holds.

As shown before, the expected value of agents also restricts the region of possible loan rates. Any loan rate must satisfy conditions both for agents and firms. Indeed, we can show  $\underline{r} < \tilde{r} < \bar{r}$  so that the region  $[\underline{r}, \tilde{r}]$  is a non-empty compact set.

**Proposition 2**  $\underline{r} < \tilde{r} < \bar{r}$  for  $q \in [0, 1]$ .

**Proof.** First, note that the above proposition holds when  $q = 0$ .

$$\begin{aligned} \lim_{q \rightarrow 0} 1 + \underline{r} &= \frac{\bar{y} - \sqrt{\bar{y}^2 - 4\alpha\bar{y}}}{2\alpha} < \lim_{q \rightarrow 0} 1 + \tilde{r} = \frac{\bar{y}}{2\alpha} \\ &< \lim_{q \rightarrow 0} 1 + \bar{r} = \frac{\bar{y} + \sqrt{\bar{y}^2 - 4\alpha\bar{y}}}{2\alpha}. \end{aligned}$$

Second, it is easy to check that  $\underline{r}$  is strictly increasing in  $c$  and  $\alpha$  for  $q \in [0, 1]$ . Then, Assumption 1 implies

$$\begin{aligned} &\alpha(2 - q)(1 + \underline{r}) \\ &= \bar{y} - qc - \sqrt{(\bar{y} - qc)^2 - 2\alpha\bar{y}(2 - q)} \\ &< \bar{y} - qc - \sqrt{(\bar{y} - q\alpha)^2 - 2\alpha\bar{y}(2 - q)} \end{aligned}$$

$$\begin{aligned}
&< \bar{y} - \frac{q\bar{y}}{4} - \sqrt{\left(\bar{y} - \frac{q\bar{y}}{4}\right)^2 - \frac{\bar{y}^2(2-q)}{2}} \\
&= \frac{\bar{y}(2-q)}{2}.
\end{aligned}$$

Now, we can show that the first inequality of the proposition holds for  $q \in (0, 1]$ .

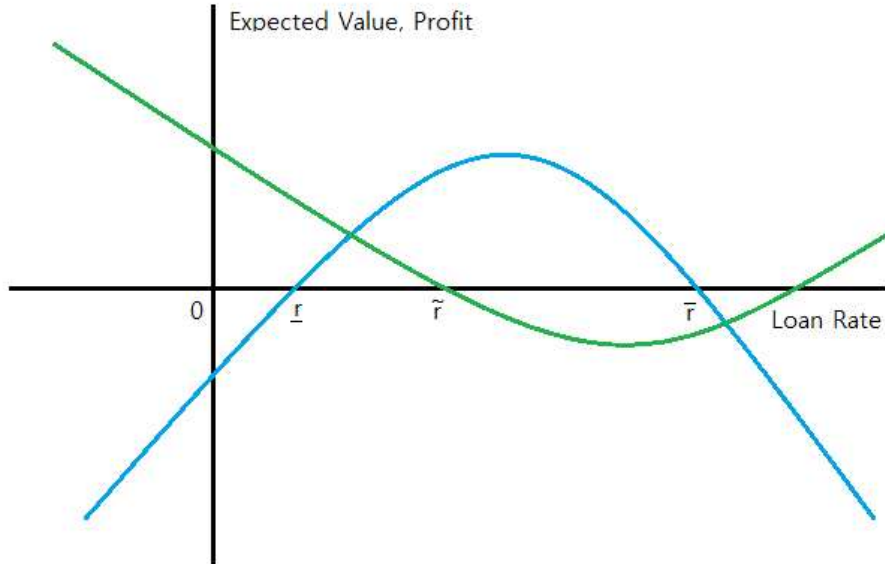
$$\begin{aligned}
1 + r &< 1 + \tilde{r} \\
\Leftrightarrow \frac{\bar{y}(2-q)}{2} &\leq \frac{\bar{y}(2-q)(1 - \sqrt{1-q})}{q} \\
\Leftrightarrow \frac{q}{2} &\leq 1 - \sqrt{1-q} \\
\Leftrightarrow \frac{q}{2}(1 + \sqrt{1-q}) &\leq q \\
\Leftrightarrow \sqrt{1-q} &\leq 1.
\end{aligned}$$

Similarly, utilizing the fact that  $\bar{r}$  is strictly decreasing in  $c$  and in  $\alpha$  with Assumption 1 proves the second inequality of the proposition for  $q \in (0, 1]$ .

$$\begin{aligned}
1 + \tilde{r} &< 1 + \bar{r} \\
\Leftrightarrow \frac{\bar{y}(2-q)(1 - \sqrt{1-q})}{q} &< \bar{y} - qc + \sqrt{(\bar{y} - qc)^2 - 2\alpha\bar{y}(2-q)} \\
\Leftrightarrow \frac{\bar{y}(2-q)(1 - \sqrt{1-q})}{q} &\leq \bar{y} - q\alpha + \sqrt{(\bar{y} - q\alpha)^2 - 2\alpha\bar{y}(2-q)} \\
\Leftrightarrow \frac{\bar{y}(2-q)(1 - \sqrt{1-q})}{q} &\leq \bar{y} - \frac{q\bar{y}}{4} + \sqrt{\left(\bar{y} - \frac{q\bar{y}}{4}\right)^2 - \frac{\bar{y}^2(2-q)}{2}} \\
\Leftrightarrow \frac{\bar{y}(2-q)(1 - \sqrt{1-q})}{q} &\leq \bar{y} \\
\Leftrightarrow (2-q)(1 - \sqrt{1-q}) &\leq q \\
\Leftrightarrow (2-q)q &\leq q(1 + \sqrt{1-q}) \\
\Leftrightarrow 1 - q &\leq \sqrt{1-q}.
\end{aligned}$$

Note that the last line holds since  $1 - q \in (0, 1]$ . ■

Figure 3: Loan Rate Satisfying Both Conditions for Agents and Financial Firms



Proposition 2 implies that the non-empty compact set  $[\underline{r}, \tilde{r}]$  is well-defined, and any loan rate in this region  $r \in [\underline{r}, \tilde{r}]$  satisfies the individual rationality conditions for agents and financial firms (Figure 3). The problem is then how many financial sectors can co-exist within this region of loan rate.

### III. Co-existence of Financial Sectors

The first thing to note is that a problem similar to limited participation arises in this model: you cannot make an incentive scheme that punishes agents strongly enough given that the agents have an option to opt out of the system. More formally,

**Proposition 3** *Suppose the verification probability is strictly less than one ( $q \in [0, 1)$ ). Let  $N$  be the largest integer that satisfies:*

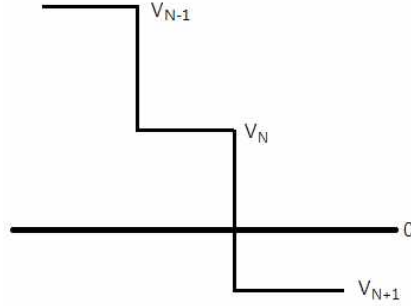
$$(i) \underline{r} \leq r_1 < r_2 < \dots < r_{N-1} < r_N \leq \tilde{r} \text{ and}$$

$$(ii) \beta m \{E[V_j(\cdot)] - E[V_{j+1}(\cdot)]\} \geq (1 - q)\alpha(1 + r_j), \text{ for } j = \{1, 2, \dots, N\}.$$

*Then, financial sector  $S_N$  cannot exist.*



Figure 4: Value of Remaining in Sector N



**Proof.** Suppose  $\beta m E[V_N(\cdot)] \geq (1-q)\alpha(1+r_N)$ . Then, Equation (5) and  $E[V_{N+1}(\cdot)] = 0$  imply that sector  $S_N$  cannot be the last sector since there can exist one more sector, say  $B_{N'}$  with  $E[V_{N'}(\cdot)] \geq 0$  and  $\beta m \{E[V_N(\cdot)] - E[V_{N'}(\cdot)]\} = (1-q)\alpha(1+r_N)$  satisfying Equation (5). This is a contradiction. On the contrary, suppose  $\beta m E[V_N(\cdot)] < (1-q)\alpha(1+r_N)$ . Then sector  $S_N$  cannot exist since Equation (5) is violated. This is a contradiction, too. ■

Proposition 3 implies that, with partial or no verification, the value function of the agents in sector  $S_{N-1}$  must satisfy  $\beta m E[V_{N-1}(\cdot)] \geq (1-q)\alpha(1+r_{N-1})$  with equality if and only if  $E[V_N(\cdot)] = 0$  so that the value of staying in the economy is sufficiently larger than that of leaving the economy. Otherwise, every agent in the last sector finds it worthwhile to tell a lie and not pay back the loan, risking leaving the economy (Figure 4). Consequently, the financial firm in sector  $S_N$  cannot recover any loans it has made. Once the financial sector  $S_N$  disappears, enough punishment is established and all the agents prefer to remain in financial sector  $S_{N-1}$  rather than leaving the economy, which allows sector  $S_{N-1}$  to exist. Note, however, Proposition 3 does not apply when full verification is in place ( $q = 1$ ). In that case Equation (5) degenerates into  $E[V_{N-1}(\cdot)] \geq E[V_N(\cdot)] \geq 0$ . This implies that  $E[V_N(\cdot)]$  can be equal to  $E[V_{N-1}(\cdot)]$  and still satisfy all the individual rationality conditions for agents so that sector  $S_N$  can co-exist with sector  $S_{N-1}$ . We will show later that there exists only one financial sector in the economy in that case.

In a stationary allocation, agent population in a specific financial sector must remain the same across periods. That is, inflow into a specific sector must be equal to outflow from that sector. This simple observation, together with the assumption that inflow into sector  $S_1$  is equal to outflow from sector  $S_{N-1}$ , leads to a stationary distribution of agent population across the financial sectors.

**Proposition 4** *In a stationary allocation,  $K_j$ , the agent population in sector  $S_j$ , can be represented by:*

$$K_j = \frac{\frac{1}{1+r_j}}{\frac{1}{1+r_1} + \dots + \frac{1}{1+r_{N-1}}}, \text{ for } j = \{1, 2, \dots, N-1\}. \quad (13)$$

**Proof.** It is easy to see that inflow into and outflow from sector  $S_1$  is represented by  $K_{N-1}\bar{y}(1+r_{N-1})m$  and  $K_1\bar{y}(1+r_1)m$ , respectively. Inflow into and outflow from sector  $S_j \in \{2, 3, \dots, N-1\}$  is represented by  $K_{j-1}\bar{y}(1+r_{j-1})m$  and  $K_j\bar{y}(1+r_j)m$ , respectively. Hence,  $K_1\bar{y}(1+r_1)m = K_2\bar{y}(1+r_2)m = \dots = K_{N-1}\bar{y}(1+r_{N-1})m$ . This leads to  $K_1(1+r_1) = K_2(1+r_2) = \dots = K_{N-1}(1+r_{N-1})$ . Solving these equations together with  $\sum_j K_j = 1$  yields the result. ■

The government agency's problem is then to construct  $(r_1, r_2, \dots, r_{N-1})$  that generates a stationary allocation as mentioned before. Following Proposition 3, it is convenient to construct the set of loan rates as if the financial sector  $S_N$  existed and then drop the last sector in the end. Note that there are multiple stationary equilibria with different numbers of financial sectors. For example, suppose that the government agency can construct a stationary allocation with  $N' \geq 2$  financial sectors. Making the number of sectors one less by removing the last sector still generates a stationary allocation. The government agency should choose an equilibrium with the maximum number of sectors. We will first cover the case with full verification.

**Proposition 5** *Suppose the verification probability is one ( $q=1$ ). Then the*

government agency can always construct a set of loan rates that generates a stationary allocation in which only one financial sector exists. In particular,  $r_1 = r_2 = \dots = r_{N-1} \in [\underline{r}, \tilde{r}]$ .

**Proof.** Pick any  $r_1 \in [\underline{r}, \tilde{r}]$ . By Equation (6),  $E[V_1(\cdot)]$  is uniquely defined for given  $r_1$ . Equation (2), when  $q=1$ , implies  $E[V_j(\cdot)] = E[V_{j+1}(\cdot)]$  for any  $j$ , which is achieved only if  $r_j = r_{j+1}$  for any  $j$ . In other words, all the loan rates across financial sectors must be the same:  $r_1 = r_2 = \dots = r_{N-1}$ . Note that individual rationality conditions for agents and financial firms are always satisfied for any  $r_j \in [\underline{r}, \tilde{r}]$  due to Proposition 2. ■

With full verification, according to Proposition 5, only one financial sector can exist in the economy if financial firms make use of full verification in which an audit takes place whenever an agent declares default. Since the audit itself imposes truth-telling, further penalty of higher loan rate next period does not have any role here. In this regard, standard debt contract model of Townsend (1979) is nicely nested in our model. Note that Proposition 2 guarantees the existence of loan rate  $r$  that generates a stationary allocation. In particular, any  $r \in [\underline{r}, \tilde{r}]$  can do the job in this case.

**Proposition 6** *Suppose the verification probability is strictly less than one ( $q \in [0, 1)$ ). Then the government agency can construct a set of loan rates  $(r_1, r_2, \dots, r_{N-1})$  that generates a stationary allocation in which  $N-1$  financial sectors co-exist if the range  $[\underline{r}, \tilde{r}]$  is sufficiently wide such that the following holds:*

$$1 + r_N \leq 1 + \tilde{r} < 1 + r_{N+1},$$

where  $r_1 = \underline{r}$  and

$$1 + r_{j+1} = \frac{\bar{y} - \sqrt{\bar{y}^2 - q\alpha(1+r_j)} \left[ 2\bar{y} + \frac{2(1-\beta)\bar{y}}{\alpha\beta m} (1-q)\alpha - q\alpha(1+r_j) \right]}{q\alpha},$$

for  $j = 1, 2, \dots, N$ . (14)

**Proof.** Without loss of generality, we can set  $r_1 = \underline{r} > 0$  since the government agency maximizes the number of co-existent financial sectors. From Equations (5) and (6) we have

$$\frac{\beta m \{ [\bar{y} - \alpha(1+r_j)]^2 - (1-q)\alpha^2(1+r_j)^2 - [\bar{y} - \alpha(1+r_{j+1})]^2 + (1-q)\alpha^2(1+r_{j+1})^2 \}}{2(1-\beta)\bar{y}}$$

$$= (1-q)\alpha(1+r_j), \text{ for } j = 1, 2, \dots, N.$$

Solving this for  $1+r_{j+1}$  in terms of  $1+r_j$  yields Equation (14). Note the set of loan rates  $(r_1, \dots, r_{N-1})$  induced by  $r_1 = \underline{r}$  and Equation (14) satisfies the incentive compatibility conditions for agents since Equation (14) incorporates Equation (5). The individual rationality conditions for agents and financial firms are also satisfied for any  $r \in [\underline{r}, \tilde{r}]$  by Proposition 2.  $1+r_N \leq 1+\tilde{r} < 1+r_{N+1}$  implies that  $N$  is the largest number such that  $r_N \leq \tilde{r}$ . Therefore,  $N-1$  financial sectors co-exist in the economy by Proposition 3. ■

Intuitively, given  $r_1 = \underline{r}$ , Equation (6) uniquely determines  $E[V_1(\cdot)]$  since  $E[V_1(\cdot)]$  is strictly decreasing in  $r$  for  $q \in [0, 1)$ . Now, Equation (5) uniquely determines  $E[V_2(\cdot)]$  and there exists a unique  $r_2$  that satisfies Equation (6). This process can be repeated until  $r_{N+1}$  is uniquely determined. It is easy to see  $\underline{r} = r_1 < r_2 < \dots < r_N < r_{N+1}$  due to Equation (6). Equation (14) formally represents this process of finding loan rates that satisfy the incentive compatibility and individual rationality conditions for agents and individual rationality conditions for financial firms.

Although  $[\underline{r}, \tilde{r}]$  is non-empty as shown in Proposition 2, Equation (5) indicates that the decrease in value can be very large, for example when  $\beta m$  is small such that  $r_2$  exceeds  $\tilde{r}$ . In this case, no financial sector can exist in the economy due to Proposition 3. Another thing to note is that a small positive shift in the loan rate can generate the same stationary allocation. Instead of  $r_1 = \underline{r}$ , consider  $r'_1 = \underline{r} + \epsilon$ ,  $\epsilon > 0$ . Then as shown in the proof of Proposition 6, Equations (5) and (6) generate consistent  $r'_2, \dots, r'_{N-1}$ . If  $r_N$  associated with

the original  $r_1$  is strictly less than  $\tilde{r}$  and  $\epsilon$  is small enough, the economy can have the same number of financial sectors.

Next, we explore the relationship between the verification probability  $q$  and the maximum number of financial sectors that can co-exist in the economy.

**Proposition 7** *Suppose the verification probability is strictly less than one ( $q \in [0, 1)$ ). As the verification probability  $q$  increases, the maximum number of financial sectors that can co-exist in the economy weakly increases.*

**Proof.** Note that  $\underline{r}$  is decreasing and  $\tilde{r}$  is increasing in  $q$ . Therefore,  $\tilde{r} - \underline{r}$  is increasing in  $q$ . Further, Equation (5) states that the gap between  $E[V_j(\cdot)]$  and  $E[V_{j+1}(\cdot)]$ , and consequently the gap between  $r_j$  and  $r_{j+1}$ , is decreasing in  $q$ . Hence, the result follows. ■

With given parameter values, Proposition 7 shows that the economy with partial verification can have at least the number of co-existent financial sectors achieved with no verification. Interestingly,  $q = 0$  simplifies Equations (6) and (7) and yields an explicit geometric form of loan rates across financial sectors, which can lead us to an easy calculation of the number of co-existent financial sectors. To see this, plug in  $q = 0$  into Equation (6) to get  $E[V_j(\cdot)]$ .

$$E[V_j(\cdot)] = \frac{1}{2(1-\beta)} [\bar{y} - 2\alpha(1+r_j)]. \quad (15)$$

Then from Equation (5),

$$1+r_{j+1} = (1+r_j) \left( \frac{1-\beta}{\beta m} + 1 \right). \quad (16)$$

Equations (15) and (16) lead to

$$1+r_j = (1+r_1) \left( \frac{1-\beta}{\beta m} + 1 \right)^{j-1}, \quad j = 1, 2, \dots, N. \quad (17)$$

Table 1: Simulation Result ( $\bar{y}=100, \alpha=20, c=10, \beta=0.9, m=0.5$ )

| $q$  | No. of Sectors | $r_1$ | $r_2$ | $r_3$ | $r_4$ | $r_{N-1}$ | $\tilde{r}$ |
|------|----------------|-------|-------|-------|-------|-----------|-------------|
| 0.00 | 2              | 0.382 | 0.689 | –     | –     | 0.689     | 1.500       |
| 0.25 | 4              | 0.355 | 0.599 | 0.891 | 1.243 | 1.243     | 1.679       |
| 0.50 | 6              | 0.333 | 0.506 | 0.705 | 0.937 | 1.530     | 1.929       |
| 0.75 | 11             | 0.314 | 0.406 | 0.506 | 0.615 | 1.808     | 2.333       |
| 0.90 | 28             | 0.305 | 0.343 | 0.382 | 0.423 | 2.477     | 2.799       |
| 1.00 | 1              | 0.298 | –     | –     | –     | –         | 4.000       |

Therefore, the number of financial sectors can be determined by the largest integer  $N-1$  such that the following holds:

$$1 + r_N = (1 + \underline{r}) \left( \frac{1 - \beta}{\beta m} + 1 \right)^{N-1} \leq \frac{\bar{y}}{2\alpha}. \quad (18)$$

In particular,

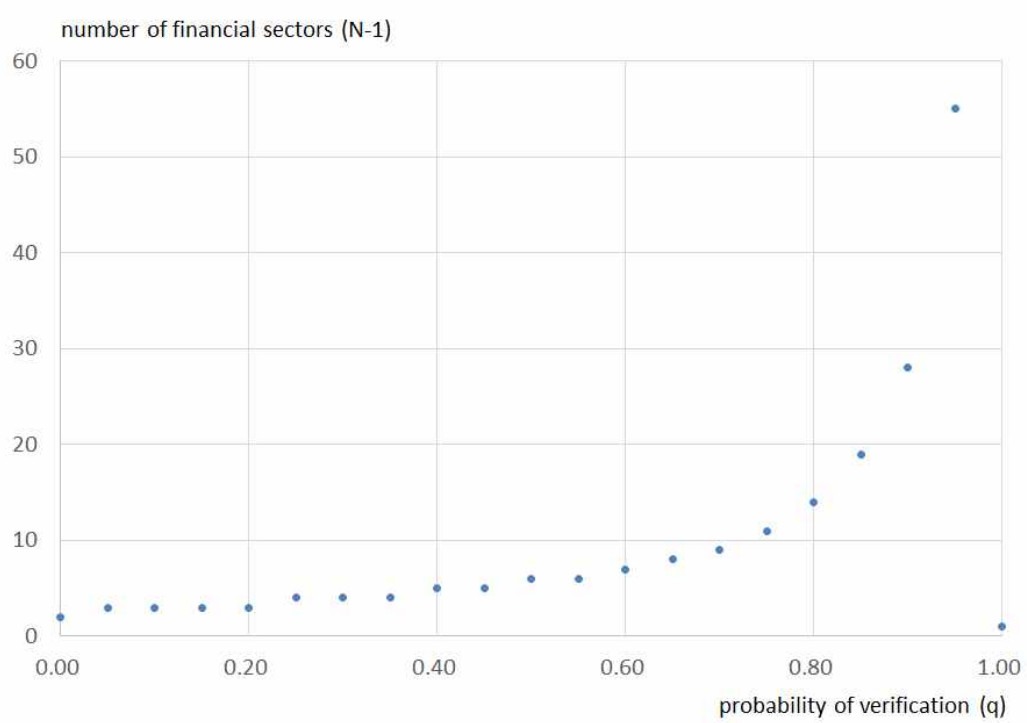
$$N-1 \leq \frac{\log(\bar{y}) - \log\left(\bar{y} - \sqrt{\bar{y}^2 - 4\alpha\bar{y}}\right)}{\log\left(1 + \frac{1 - \beta}{\beta m}\right)}. \quad (19)$$

A simulation would clearly show the results so far. We present a simulation result with the parameter values set as  $\bar{y}=100, \alpha=20, c=10, \beta=0.9, m=0.5$ . Note that Equation (19) tells us that two financial sectors can co-exist when  $q=0$ . Observe that the number of financial sectors in the economy weakly increases as  $q \in [0, 1)$  gets larger and that when  $q=1$  there can exist only one financial sector (Table 1 and Figure 5).<sup>10</sup> In the latter case the government agency can generate a stationary allocation by choosing any  $r \in [0.298, 4.000]$ . The number of co-existent financial sectors presented in the simulation represents the maximum number the government agency can achieve by

<sup>10</sup> The number of co-existent financial sectors, hence, does not monotonically increase in  $q$ . In terms of loan rate differences across financial sectors, however, the monotonicity is retained. Equation (5), indeed, states that the loan rate differences offered in sector  $S_j$  and sector  $S_{j+1}$  monotonically decreases to zero as  $q$  increases to one.

Figure 5: Number of Co-existent Financial Sectors

$$(\bar{y} = 100, \alpha = 20, c = 10, \beta = 0.9, m = 0.5)$$



aligning the loan rates in such a way to satisfy the incentive compatibility and individual rationality conditions.

Some parameter values fail to generate any financial sectors. With  $\bar{y} = 100$ ,  $\alpha = 24.5$ ,  $c = 10$ ,  $\beta = 0.9$ ,  $m = 0.5$ , no financial sectors can exist when  $q = 0$ . Although  $[\underline{r}, \tilde{r}] = [0.752, 1.041]$  is not empty, setting  $r_1 = \underline{r} = 0.752$  leads to  $r_2 = 1.142 > \tilde{r}$ . In this situation, the incentive compatibility conditions for agents in financial sector  $S_1$ , as given in Equation (5), cannot be satisfied and all the agents in sector  $S_1$  would not pay back their loans. Thus, the profit of the financial firm  $B_1$  should be negative and sector  $S_1$  cannot exist. Note that Proposition 3 precisely implies this result.

Below, we provide some further comparative statics results and discuss their implications.

**Proposition 8** *Suppose the verification probability is strictly less than one ( $q \in [0, 1)$ ). The maximum number of financial sectors that can co-exist in the economy weakly increases with larger  $\bar{y}$ , smaller  $\alpha$ , smaller  $c$ , and larger  $m$ .*

**Proof.** Proofs are similar to that of Proposition 7, and hence omitted here. ■

Proposition 8 states that an increase in the maximum achievable output from a project  $\bar{y}$  (recall that  $y_i \sim U[0, \bar{y}]$ ) makes room for more financial sectors to co-exist in the economy. It widens the range  $[\underline{r}, \tilde{r}]$  while decreasing the gap between any adjacent loan rates,<sup>11)</sup> which forms a necessary condition for more financial sectors to co-exist. A decrease in the initial amount of loan  $\alpha$  brings the same result.

The government agency might want to set up a relief fund so as to prevent some portion of agents who would fail to repay the debt from falling from financial sector  $S_j$  to  $S_{j+1}$ . This should be similar, in effect, to a decrease in  $m$ , which would lead to, maybe unintendedly, fewer financial sectors co-existing. Alleviating punishment by setting up a relief fund in this way must be compensated or balanced with heavier punishment so that the gap of expected values across financial sectors is widened to keep the truth-telling mechanism in place. According to Proposition 7, the government agency should instead invest in increasing the verification technology  $q$ , if possible, to increase the number of co-existent financial sectors by narrowing the gaps of loan rates across financial sectors.

Now we turn our discussion to efficiency. It seems obvious that no verification regime brings the highest efficiency: the same production as with verification, but without the associated costs.<sup>12)</sup>

---

11) To see this, take the partial derivative of Equation (14) with respect to  $\bar{y}$ .

12) Note, however, that a direct comparison of efficiency between full and partial verification does not yield a one-way result. Full verification can be more or less efficient than partial verification depending on the specific loan rate set in the former. For example, under the parameter values specified in (Table 1), the total cost of verification is 0.856 when  $q = 0.25$  and 2.754 when  $q = 0.75$ , while the total cost can be in  $[2.597, 10.000]$  when  $q = 1$  depending on the specific loan rate  $r \in [0.298, 4.000]$  imposed by the financial firm.



**Proposition 9** *No verification is more efficient than full or partial verification in the sense that all three methods of verification yield the same production every period but no verification does not incur any amount of verification cost.*

So far we have focused on the construction of a set of loan rates that generates stationary allocation with as many financial sectors as possible. We will now discuss an environment in which the government agency is taken out and each financial firm is allowed to freely choose the loan rate to maximize its profit. If financial firms have market power, differentiation motives arise across financial firms. In the context of this paper, financial firms would have incentives to charge different loan rates in the process of maximizing profits.

To see this, suppose first that only one financial firm exists in the economy. Note that its profit  $\pi_j$  as represented in Equation (9) is increasing in  $r_j$  within the whole range of  $[\underline{r}, \tilde{r}]$ . Therefore, the monopoly will choose to operate in the last sector  $S_{N-1}$  charging the loan rate of  $r_{N-1}$  set in the government agency case. The monopoly, however, can do a little bit better. The government agency chooses  $r_1 = \underline{r}$  first and constructs  $r_j$  in such a way that the incentive compatibility and individual rationality conditions of agents are satisfied. Alternatively, the monopoly can choose the loan rate  $r_{N-1}$  backwards by imposing  $E[V_N(\cdot)] = 0$  under the restriction of Equation (5),  $\beta m \{E[V_{N-1}(\cdot)] - E[V_N(\cdot)]\} = (1-q)\alpha(1+r_{N-1})$ . An example can be useful to illustrate this. Suppose that the parameter values are set as the same as in (Table 1) except  $q$  is fixed at zero:  $\bar{y} = 100$ ,  $\alpha = 20$ ,  $c = 10$ ,  $\beta = 0.9$ ,  $m = 0.5$ , and  $q = 0$ . Consider a monopoly setting the loan rate. Plug  $E[V_N(\cdot)] = 0$  and  $q = 0$  into Equations (5) and (6) to get:

$$\begin{aligned} E[V_{N-1}(\cdot)] &= \frac{1}{\beta m} \alpha (1 + r_{N-1}) \\ &= \frac{1}{2(1-\beta)} [\bar{y} - 2\alpha(1 + r_{N-1})]. \end{aligned} \quad (20)$$

This yields

$$1 + r_{N-1} = \frac{\beta m \bar{y}}{2\alpha [1 - \beta(1 - m)]}. \quad (21)$$

Given the set of parameter values above, the monopoly sets  $r_1^* = 1.045$  to have 4.174 profit (per period). Recall that the government agency has constructed the set of interest rates  $(r'_1, r'_2) = (0.382, 0.689)$ . If the monopoly chooses  $r_1^* = r'_2 = 0.689$ , the corresponding profit shrinks to 2.370. What if there exists one more financial firm? In this duopoly situation, both financial firms set  $r_1^* = 1.045$ , each enjoying 2.087 profit. If financial firms form two sectors by charging  $(r_1^*, r_2^*) = (0.674, 1.045)$  so that the gap between  $r_1^*$  and  $r_2^*$  is just enough to satisfy the agents' incentive compatibility conditions, the corresponding profit is 1.247 and 1.878 for sector  $S_1^*$  and  $S_2^*$ , respectively.<sup>13)</sup> A situation with three firms does not change the equilibrium much. All three firms choose to operate in the same sector  $S_1^*$ , charging loan rate  $r_1^* = 1.045$  and share the total profit of 4.174 equally to have 1.391 each. Now consider the case with four financial firms. If all firms choose to operate in the same sector, each enjoys  $4.174/4 = 1.043$  profit. Now suppose one firm deviates to operate in a different sector offering loan rate  $r_1^* = 0.674$ . It will have increased profit 1.247 with population  $K_1 = 0.55$  while the other three operating in  $S_2^*$  offering  $r_2^* = 1.045$  will have 0.626 profit each. Does one of the other three financial firms operating in sector  $S_2^*$  still have an incentive to deviate to operate in sector  $S_1^*$ ? No, because doing so will decrease the profit from 0.626 to  $1.247/2 = 0.624$ . Therefore, an equilibrium will be one firm operating in sector  $S_1^*$  and the other three operating in  $S_2^*$ . This means that profit-maximizing financial firms also generate multiple financial sectors in equilibrium. (Table 2)

---

13) Note that  $(r_1^*, r_2^*)$  is the set of largest loan rates that satisfies the incentive compatibility condition as given in Equation (5) and indeed maximizes the profit under the condition that these two financial firms choose to offer different loan rates. To see this, note  $\partial \Pi_1 / \partial r_1^* > 0$  at given  $r_1^*$  and  $r_2^*$ , which implies that the financial firm operating in sector  $S_1^*$  must increase the loan rate to the extent that the incentive compatibility condition permits to maximize its profit.

Table 2: Profit Maximization and Number of Co-existent Financial Sectors

$$(\bar{y} = 100, \alpha = 20, c = 10, \beta = 0.9, m = 0.5, q = 0)$$

| No. of Firms | $r_1^*$ | $K_1^*$ | Profit per Firm in $S_1^*$ | $r_2^*$ | $K_2^*$ | Profit per Firm in $S_2^*$ |
|--------------|---------|---------|----------------------------|---------|---------|----------------------------|
| One          | 1.045   | 1.00    | 4.174                      | –       | –       | –                          |
| Two          | 1.045   | 1.00    | 2.087                      | –       | –       | –                          |
| Three        | 1.045   | 1.00    | 1.391                      | –       | –       | –                          |
| Four         | 0.674   | 0.55    | 1.247                      | 1.045   | 0.45    | 0.626                      |

illustrates the equilibrium loan rates, populations and profits across financial sectors.

It is interesting to find, in this example, that financial firms' strategies in setting the loan rates are effectively finite. Equation (5) picks out a finite set of loan rates that can be charged by the financial firms. This set of loan rates, being selected from a profit maximization perspective given the incentive compatibility and individual rationality conditions of the agents, is different from the set chosen by the government agency. The difference, in particular, arises from the fact that profit-maximizing firms impose  $E[V_N(\cdot)] = 0$ .

Of course, financial firms face infinitely repeated games in choosing loan rates. Financial firms can collude by using trigger strategies so that only sector  $S_2$  exists in the stationary equilibrium. But as the number of financial firms entering the economy grows, the payoff from deviation gets bigger and eventually breaks the collusion. Summing up, profit maximizing financial firms can construct financial sectors that charge discretely different loan rates provided that the number of financial firms is sufficiently large.

This example makes room for government intervention to control the loan rate. Suppose the government agency introduces a loan rate ceiling at somewhere between  $r'_2$  and  $r_2^*$  and issues a sufficient number of licenses to financial firms. Then the economy has two financial sectors with lower levels of loan rates than the case with no ceiling imposed. As a consequence, agents' expected values across the financial sectors increase while the profits of financial firms decrease.

## IV. Conclusion

In this paper, we have studied a specific form of debt contract — a standard one with a given verification probability that can be set anywhere between zero and one — in a dynamic environment. We have focused on the government agency's problem in building a stationary and non-symmetric allocation in which financial firms offer different loan rates, and have shown that partial verification materializes the co-existence of different loan rates offered by different financial sectors since discretely different loan rates constitute an elaborate truth-telling mechanism. This mechanism does not require the financial firms to verify even if the borrower declares bankruptcy, and therefore is more efficient than a standard debt contract à la Townsend (1979) in terms of verification cost. Exploiting the fact that the loan rates offered across financial sectors should be well aligned in order to satisfy the incentive compatibility conditions of borrowers, we have derived the maximum number of financial sectors that can co-exist in the economy. The differences in loan rates across financial sectors are shown to be affected by, among other factors, the verification probability: As verification probability increases the loan rate differences across financial sectors decreases. The government, therefore, may want to narrow the loan rate differences by increasing the verification probability if possible. This measure brings down the lowest loan rate offered in the economy, too. It is, however, not without cost in the context of our model. Associated verification cost may increase while the total production remains at the same level. The highest loan rate offered in the economy also increases.

We also have illustrated that the results derived with the government agency basically continue to hold when the government agency is taken out and financial firms are allowed to freely choose their loan rates to maximize their profits. If only one financial firm exists, the monopoly chooses its profit maximizing loan rate in such a way that the agent who fails to pay back the loan cannot have a second chance, i.e. the agent must leave the economy.<sup>14)</sup>

As the number of financial firms increases, profit-maximizing financial firms

---

14) As already shown in Proposition 6, this solution requires that the range of loan rates that satisfy the incentive compatibility and individual rationality conditions for agents and firms must be sufficiently wide to prevent a kind of limited participation problem.

start to offer different loan rates. We have noted that the loan rates set by the government agency and profit-maximizing financial firms are different, which creates room for government intervention in the financial market, for example, in the form of setting a loan rate ceiling. We must admit, of course, that this part that incorporates profit maximizing financial firms needs more formal treatment to arrive at more general results.

## References

- Albuquerque, R. and H. A. Hopenhayn (2004), "Optimal Lending Contracts and Firm Dynamics," *Review of Economic Studies*, Vol. 71(2), pp. 285-315.
- Banerjee, A. V. and E. Duflo (2011), "The (not so Simple) Economics of Lending to the Poor," *Lecture Note*, Department of Economics, MIT.
- Chang, C. (1990), "The Dynamic Structure of Optimal Debt Contracts," *Journal of Economic Theory*, Vol. 52(1), pp. 68-86.
- Christiano, L. J., R. Motto and M. Rostagno (2014), "Risk Shocks," *American Economic Review*, Vol. 104(1), pp. 27-65.
- Cooley, T., R. Marimon and V. Quadrini (2004), "Aggregate Consequences of Limited Contract Enforceability," *Journal of Political Economy*, Vol. 112(4), pp. 817-847.
- Ghosh, P., D. Mookherjee and D. Ray (1999), "Credit Rationing in Developing Countries: An Overview of the Theory," *mimeo*.
- Monnet, C. and E. Quintin (2005), "Optimal Contracts in a Dynamic Costly State Verification Model," *Economic Theory*, Vol. 26(4), pp. 867-885.
- Mookherjee, D. and I. Png (1989), "Optimal Auditing, Insurance, and Redistribution," *Quarterly Journal of Economics*, Vol. 104(2), pp. 399-415.
- Stiglitz, J. E. and A. Weiss (1981), "Credit Rationing in Markets with Imperfect Information," *American Economic Review*, Vol. 71(3), pp. 393-410.
- Townsend, R. M. (1979), "Optimal Contracts and Competitive Markets with Costly State Verification," *Journal of Economic Theory*, Vol. 21(2), pp. 265-293.

## <Abstract in Korean>

# 금융권별 대출금리 격차에 대한 이론적 분석

김병기\*, 민준규\*\*

이 소고는 기제고안(mechanism design)이론을 통해 금융권별 대출금리 격차가 존재하는 이유를 이론적으로 도출하였다. 기존 문헌에서는 대출금리 격차의 원인을 단순히 차주의 부도확률 차이에 기인하는 것으로 이해하여 온 반면, 이 소고에서는 대출금리 격차 그 자체가 원리금 상환능력에 대한 은행과 차입자 사이의 정보비대칭성을 효율적으로 완화하는 기제가 될 수 있다는 사실을 이론적으로 증명하였다. 즉, 이번 기에 특정 금융권으로부터 일정 금리로 자금을 빌린 차입자가 원리금 상환을 거부할 경우 다음 기부터는 보다 높은 금리를 요구하는 다른 금융권에서 자금을 차입해야 한다면 이러한 환경이 차입자로 하여금 상환능력에 대해 진실을 말하는 기제로 작동할 수 있음을 보였다. 특히, 기존 문헌에서 차입자의 거짓부도를 방지하기 위한 기제로 많이 제시되어 온 은행의 사후검사가 어렵거나 불가능한 상황에서도 금융권간 대출금리 격차를 잘 설계하면 차입자가 상환능력이 없는 것으로 가장함으로써 얻을 수 있는 기대수익이 진실을 말함으로써 얻을 수 있는 기대수익을 하회하도록 하는 것이 가능하다는 결과를 제시하였다. 또한 이 소고에서는 차입자가 진실을 말하도록 유도하기 위해서는 금융권간 대출금리 격차가 잘 설계되어야 한다는 점을 이용하여 경제 내에 각각 다른 대출금리를 설정하고 있는 금융권이 최대 몇 개 공존할 수 있는지도 도출하였다. 구체적으로 경제 내에 공존 가능한 금융권의 개수가 금융기관의 사후검사 수행 확률, 차입금을 활용한 프로젝트의 기대수익, 차입원금의 규모, 차입자의 부도선언 시 금융거래 중단 확률 등에 달려 있음을 보였다.

핵심 주제어: 대출시장, 기제고안(메카니즘 디자인, mechanism design), 금융권별 금리격차, 거짓부도 방지 유인 설계, 공존 가능한 금융권의 수

JEL Classification: D82, G21, G23

\* 한국은행 통화정책국 정책제도연구팀장 (전화: 02-759-4941, E-mail: bkkim@bok.or.kr)

\*\* 한국은행 경제연구원 북한경제연구실장 (전화: 02-759-5480, E-mail: jgmin@bok.or.kr)

## BOK 경제연구 발간목록

한국은행 경제연구원에서는 Working Paper인 『BOK 경제연구』를 수시로 발간하고 있습니다. 『BOK 경제연구』는 주요 경제 현상 및 정책 효과에 대한 직관적 설명 뿐 아니라 깊이 있는 이론 또는 실증 분석을 제공함으로써 엄밀한 논증에 초점을 두는 학술논문 형태의 연구이며 한국은행 직원 및 한국은행 연구용역사업의 연구 결과물이 수록되고 있습니다. 『BOK 경제연구』는 한국은행 경제연구원 홈페이지(<http://imer.bok.or.kr>)에서 다운로드하여 보실 수 있습니다.

- |          |  |   |
|----------|--|---|
| 제2014 -1 | Network Indicators for Monitoring Intraday Liquidity in BOK-Wire+  | Seungjin Baek · Kimmo Soram ki · Jaeho Yoon                   |
| 2        | 중소기업에 대한 신용정책 효과   | 정호성 · 임호성   |
| 3        | 경제충격 효과의 산업간 공행성 분석  | 황선웅 · 민성환 · 신동현 · 김기호   |
| 4        | 서비스업 발전을 통한 내외수 균형성장: 기대효과 및 리스크   | 김승원 · 황광명   |
| 5        | Cross-country-heterogeneous and Time-varying Effects of Unconventional Monetary Policies in AEs on Portfolio Inflows to EMEs     | Kyoungsoo Yoon · Christophe Hurlin                            |
| 6        | 인터넷뱅킹, 결제성예금 및 은행 수익성과의 관계 분석  | 이동규 · 전봉걸   |
| 7        | Dissecting Foreign Bank Lending Behavior During the 2008-2009 Crisis   | Moon Jung Choi · Eva Gutierrez · Maria Soledad Martinez Peria |
| 8        | The Impact of Foreign Banks on Monetary Policy Transmission during the Global Financial Crisis of 2008-2009: Evidence from Korea | Bang Nam Jeon · Hosung Lim · Ji Wu                            |
| 9        | Welfare Cost of Business Cycles in Economies with Individual Consumption Risk  | Martin Ellison · Thomas J. Sargent                            |
| 10       | Investor Trading Behavior Around the Time of Geopolitical Risk Events: Evidence from South Korea                                 | Young Han Kim · Hosung Jung                                   |
| 11       | Imported-Inputs Channel of Exchange Rate Pass-Through: Evidence from Korean Firm-Level Pricing Survey                            | Jae Bin Ahn · Chang-Gui Park                                  |
-



---

|          |  |   |
|----------|--|---|
| 제2014-12 | 비대칭 금리기간구조에 대한 실증분석  | 김기호   |
| 13       | The Effects of Globalization on Macroeconomic Dynamics in a Trade-Dependent Economy: the Case of Korea                 | Fabio Milani · Sung Ho Park                     |
| 14       | 국제 포트폴리오투자 행태 분석: 채권-주식 투자자금간 상호관계를 중심으로   | 이주용 · 김근영                                       |
| 15       | 북한 경제의 추격 성장 가능성과 정책 선택 시나리오   | 이근 · 최지영  |
| 16       | Mapping Korea's International Linkages using Generalised Connectedness Measures  | Hail Park · Yongcheol Shin                      |
| 17       | 국제자본이동 하에서 환율신축성과 경상수지 조정: 국가패널 분석   | 김근영   |
| 18       | 외국인 투자자가 외환시장과 주식시장 간 유동성 동행화에 미치는 영향  | 김준한 · 이지은                                       |
| 19       | Forecasting the Term Structure of Government Bond Yields Using Credit Spreads and Structural Breaks                    | Azamat Abdymomunov · Kyu Ho Kang · Ki Jeong Kim |
| 20       | Impact of Demographic Change upon the Sustainability of Fiscal Policy  | Youngguk Kim · Myoung Chul Kim · Seongyong Im   |
| 21       | The Impact of Population Aging on the Countercyclical Fiscal Stance in Korea, with a Focus on the Automatic Stabilizer | Tae-Jeong Kim · Mihye Lee · Robert Dekle        |
| 22       | 미 연준과 유럽중앙은행의 비전통적 통화정책 수행원칙에 관한 고찰  | 김병기 · 김진일                                       |
| 23       | 우리나라 일반인의 인플레이션 기대 형성 행태 분석  | 이한규 · 최진호                                       |

---

---

|          |  |  |
|----------|--|--|
| 제2014-24 | Nonlinearity in Nexus between Working Hours and Productivity                           | Dongyeol Lee · Hyunjoon Lim  |
| 25       | Strategies for Reforming Korea's Labor Market to Foster Growth                         | Mai Dao · Davide Furceri · Jisoo Hwang · Meeyeon Kim · Tae-Jeong Kim |
| 26       | 글로벌 금융위기 이후 성장잠재력 확충: 2014 한국은행 국제컨퍼런스 결과보고서   | 한국은행 경제연구원   |
| 27       | 인구구조 변화가 경제성장률에 미치는 영향: 자본이동의 역할에 대한 논의를 중심으로  | 손종철  |
| 28       | Safe Assets  | Robert J. Barro  |
| 29       | 확장된 실업지표를 이용한 우리나라 노동시장에서의 이력현상 분석   | 김현학 · 황광명  |
| 30       | Entropy of Global Financial Linkages   | Daeyup Lee   |
| 31       | International Currencies Past, Present and Future: Two Views from Economic History     | Barry Eichengreen  |
| 32       | 금융체제 이행 및 통합 사례: 남북한 금융통합에 대한 시사점  | 김병연  |
| 33       | Measuring Price-Level Uncertainty and Instability in the U.S., 1850-2012               | Timothy Cogley · Thomas J. Sargent                                   |
| 34       | 고용보호제도가 노동시장 이원화 및 노동생산성에 미치는 영향   | 김승원  |
| 35       | 해외충격시 외화예금의 역할 : 주요 신흥국 신용스프레드에 미치는 영향을 중심으로   | 정호성 · 우준명  |
| 36       | 실업률을 고려한 최적 통화정책 분석  | 김인수 · 이명수  |
| 37       | 우리나라 무역거래의 결제통화 결정요인 분석  | 황광명 · 김경민 · 노충식 · 김미진  |
| 38       | Global Liquidity Transmission to Emerging Market Economies, and Their Policy Responses | Woon Gyu Choi · Taesu Kang · Geun-Young Kim · Byongju Lee            |

---

---

|          |  |   |
|----------|--|---|
| 제2015 -1 | 글로벌 금융위기 이후 주요국<br>통화정책 운영체계의 변화   | 김병기 · 김인수   |
| 2        | 미국 장기시장금리 변동이 우리나라<br>금리기간구조에 미치는 영향 분석 및<br>정책적 시사점   | 강규호 · 오형석   |
| 3        | 직간접 무역연계성을 통한 해외충격의<br>우리나라 수출입 파급효과 분석  | 최문정 · 김근영   |
| 4        | 통화정책 효과의 지역적 차이  | 김기호   |
| 5        | 수입중간재의 비용효과를 고려한<br>환율변동과 수출가격 간의 관계   | 김경민   |
| 6        | 중앙은행의 정책금리 발표가<br>주식시장 유동성에 미치는 영향   | 이지은   |
| 7        | 은행 건전성지표의 변동요인과<br>거시건전성 규제의 영향  | 강종구   |
| 8        | Price Discovery and Foreign Participation<br>in The Republic of Korea's<br>Government Bond Futures<br>and Cash Markets | Jaehun Choi · Hosung Lim ·<br>Rogelio Jr. Mercado ·<br>Cyn-Young Park |
| 9        | 규제가 노동생산성에 미치는 영향:<br>한국의 산업패널 자료를 이용한 실증분석  | 이동렬 · 최종일 · 이종한   |
| 10       | 인구 고령화와 정년연장 연구<br>(세대 간 중첩모형(OLG)을 이용한 정량 분석)   | 홍재화 · 강태수   |
| 11       | 예측조합 및 밀도함수에 의한<br>소비자물가 상승률 전망  | 김현학   |
| 12       | 인플레이션 동학과 통화정책   | 우준명   |
| 13       | Failure Risk and the Cross-Section<br>of Hedge Fund Returns  | Jung-Min Kim  |
| 14       | Global Liquidity and Commodity Prices  | Hyunju Kang ·<br>Bok-Keun Yu ·<br>Jongmin Yu                          |
| 15       | Foreign Ownership, Legal System<br>and Stock Market Liquidity  | Jieun Lee · Kee H. Chung  |

---

---

|          |  |   |
|----------|--|---|
| 제2015-16 | 바젤Ⅲ 은행 경기대응완충자본 규제의<br>기준지표에 대한 연구   | 서현덕 · 이정연   |
| 17       | 우리나라 대출 수요와 공급의 변동요인 분석  | 강종구 · 임호성   |
| 18       | 북한 인구구조의 변화 추이와 시사점  | 최지영   |
| 19       | Entry of Non-financial Firms and Competition<br>in the Retail Payments Market  | Jooyong Jun   |
| 20       | Monetary Policy Regime Change<br>and Regional Inflation Dynamics:<br>Looking through the Lens of<br>Sector-Level Data for Korea            | Chi-Young Choi ·<br>Joo Yong Lee ·<br>Roisin O'Sullivan             |
| 21       | Costs of Foreign Capital Flows<br>in Emerging Market Economies:<br>Unexpected Economic Growth<br>and Increased Financial Market Volatility | Kyoungsoo Yoon ·<br>Jayoung Kim                                     |
| 22       | 글로벌 금리 정상화와 통화정책 과제:<br>2015년 한국은행 국제컨퍼런스 결과보고서  | 한국은행 경제연구원  |
| 23       | The Effects of Global Liquidity<br>on Global Imbalances  | Marie-Louise DJIBENOU-KRE ·<br>Hail Park                            |
| 24       | 실물경기를 고려한 내재 유동성 측정  | 우준명 · 이지은   |
| 25       | Deflation and Monetary Policy  | Barry Eichengreen   |
| 26       | Macroeconomic Shocks<br>and Dynamics of Labor Markets in Korea   | Tae Bong Kim ·<br>Hangyu Lee  |
| 27       | Reference Rates and Monetary Policy<br>Effectiveness in Korea  | Heung Soon Jung ·<br>Dong Jin Lee ·<br>Tae Hyo Gwon ·<br>Se Jin Yun |
| 28       | Energy Efficiency and Firm Growth  | Bongseok Choi ·<br>Wooyoung Park ·<br>Bok-Keun Yu                   |
| 29       | An Analysis of Trade Patterns<br>in East Asia and the Effects of<br>the Real Exchange Rate Movements                                       | Moon Jung Choi ·<br>Geun-Young Kim ·<br>Joo Yong Lee                |
| 30       | Forecasting Financial Stress Indices in<br>Korea: A Factor Model Approach  | Hyeongwoo Kim ·<br>Hyun Hak Kim ·<br>Wen Shi                        |

---

---

|       |    |   |   |
|-------|----|---|---|
| 제2016 | -1 | The Spillover Effects of U.S. Monetary Policy on Emerging Market Economies: Breaks, Asymmetries and Fundamentals          | Geun-Young Kim ·<br>Hail Park ·<br>Peter Tillmann |
|       | 2  | Pass-Through of Imported Input Prices to Domestic Producer Prices: Evidence from Sector-Level Data                        | JaeBin Ahn ·<br>Chang-Gui Park ·<br>Chanho Park   |
|       | 3  | Spillovers from U.S. Unconventional Monetary Policy and Its Normalization to Emerging Markets: A Capital Flow Perspective | Sangwon Suh ·<br>Byung-Soo Koo                    |
|       | 4  | Stock Returns and Mutual Fund Flows in the Korean Financial Market: A System Approach                                     | Jaebeom Kim ·<br>Jung-Min Kim                     |
|       | 5  | 정책금리 변동이 성별·세대별 고용률에 미치는 영향   | 정성엽   |
|       | 6  | From Firm-level Imports to Aggregate Productivity: Evidence from Korean Manufacturing Firms Data                          | JaeBin Ahn ·<br>Moon Jung Choi                    |
|       | 7  | 자유무역협정(FTA)이 한국 기업의 기업내 무역에 미친 효과   | 전봉걸 · 김은숙 · 이주용                                   |
|       | 8  | The Relation Between Monetary and Macroprudential Policy  | Jong Ku Kang                                      |
|       | 9  | 조세피난처 투자자가 투자 기업 및 주식 시장에 미치는 영향  | 정호성 · 김순호   |
|       | 10 | 주택실거래 자료를 이용한 주택부문 거시 건전성 정책 효과 분석  | 정호성 · 이지은   |
|       | 11 | Does Intra-Regional Trade Matter in Regional Stock Markets?: New Evidence from Asia-Pacific Region                        | Sei-Wan Kim ·<br>Moon Jung Choi                   |
|       | 12 | Liability, Information, and Anti-fraud Investment in a Layered Retail Payment Structure                                   | Kyoung-Soo Yoon ·<br>Jooyong Jun                  |
|       | 13 | Testing the Labor Market Dualism in Korea   | Sungyup Chung ·<br>Sunyoung Jung                  |
|       | 14 | 북한 이중경제 사회계정행렬 추정을 통한 비공식부문 분석  | 최지영   |

---

---

|           |   |  |
|-----------|---|--|
| 제 2016-15 | Divergent EME Responses to Global and Domestic Monetary Policy Shocks       | Woon Gyu Choi ·<br>Byongju Lee ·<br>Taesu Kang ·<br>Geun-Young Kim |
| 16        | Loan Rate Differences across Financial Sectors: A Mechanism Design Approach | Byoung-Ki Kim ·<br>Jun Gyu Min                                     |

---