Trend Growth Shocks and Asset Prices

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Trend Growth Shocks and Asset Prices

This paper addresses the link between shocks to productivity trend growth and long-run consumption risk in a production economy model with recursive utility. Quantifying trend growth shocks, I find that persistent fluctuations in trend growth are the key driver of sizable long-run consumption risk. I compare this result to two conventional assumptions on a productivity process: 1) a deterministic trend with a cycle and 2) a random walk with drift. Persistent trend growth shocks generate larger long-run consumption risk than both highly persistent cycle shocks and random walk shocks. As a result, agents in the face of the trend growth shocks tend to save more and demand a higher equity premium. In addition, fluctuations in aggregate productivity growth is largely attributable to movements in trend growth.

**Keywords:** Long-run consumption risk, stochastic trend growth, equity premium, production economy, exact initial Kalman filter

**JEL Classification:** E21, E23, E30, G12
I. Introduction

A number of macroeconomic and consumption-based asset pricing models have regarded consumption growth as an independent and identically distributed (i.i.d. hereafter) random variable (e.g., Hall 1979; Campbell and Cochrane 1999; Bekaert and Engstrom 2017). However, recent empirical works have questioned this long-standing view by presenting evidence for persistent shocks to expected consumption growth, which is referred to as long-run consumption risk (Nakamura et al. 2017; Schorfheide et al. 2018). As a graphical example, the solid line of figure 1 shows the evolution of the conditional expectation of U.S. consumption growth for the 1955–2014 period.\(^1\) Persistent variation in expected consumption growth is evident in the figure, which accounts for a large fraction of consumption growth volatility (roughly 38%).

The empirical evidence rationalizes the adoption of Epstein and Zin (1989) (EZ hereafter) preferences, the resulting stochastic discount factor from which reflects long-run consumption risk. Bansal and Yaron (2004) show that long-run risk (LRR hereafter) in combination with EZ preferences can unravel major asset pricing puzzles in an exchange economy: the equity premium puzzle (Mehra and Prescott 1985), the risk-free rate puzzle (Weil 1989), and the volatility puzzle (LeRoy and Porter 1981; Shiller 1981). However, a quantitatively consistent explanation for sizable long-run consumption risk in a production economy has not been extensively put forward yet.

How do we such sizable long-run consumption risk justify in a production economy? In this paper, I show that the answer to this question hinges on the type of productivity specification. Macroeconomists have traditionally relied on either or both of the two types of a productivity shock: 1) a transitory shock (e.g., a deterministic trend and a

\(^{1}\) The conditional expectation of U.S. consumption growth is estimated based on a state-space model for consumption growth specified by Bansal and Yaron (2004) under a homoscedasticity assumption. See Appendix A for the details on parameter estimates.
Figure 1. Consumption growth and its conditional expectation

Notes: This figure plots demeaned consumption growth (dotted line) and expected demeaned consumption growth (solid line). I use the Kalman smoother to obtain the unobserved expected consumption growth. Data are real, per capita consumption growth from 1955:1–2014:4.

cycle) and 2) a permanent shock that is i.i.d.(e.g., random walk with drift). I demonstrate that a persistent shock to trend (permanent component) growth contributes greatly to sizable long-run consumption risk, whereas neither productivity shock has a significant contribution. This result is consistent with the permanent income hypothesis (PIH).

Based on the PIH point of view, my underlying premise is that an economy characterized by a deterministic productivity trend makes agents less concerned about future income risk than an economy characterized by a stochastic trend. Agents in the economy are aware that they would end up following the deterministic income path, which may cause the expected future consumption to be less volatile (i.e., smaller magnitude of the consumption LRR). On the other hand, in the economy characterized by the stochastic trend in productivity, agents would expect their future
income level to be liable to change instantaneously, and this produces sizable LRR in consumption. The consumption LRR would become larger when agents face persistent trend growth in productivity than under i.i.d. trend growth in productivity. This is because once a non-zero persistent growth shock occurs, they anticipate the income level changing gradually in the future.

I build a parsimonious model economy in which a representative agent has EZ preferences and utilizes a neoclassical production technology subject to capital adjustment costs and aggregate productivity shocks. EZ preferences have a well-known feature: separation of the elasticity of intertemporal substitution (EIS) from the relative risk aversion (RRA). This recursive utility is appropriate to capture the effect of news about a future path of consumption, if consumption growth has a persistent component (e.g., Kocherlakota 1990; Cochrane 2008). To examine the prediction of the PIH, I propose an I(1) productivity process, which includes four special specifications: 1) a stochastic trend, the growth of which is persistent, and a cycle; 2) the stochastic trend, the growth of which is persistent, and a pure random walk; 3) a deterministic trend and a cycle; and 4) a random walk with drift.

The first contribution is to provide direct empirical evidence for persistent trend growth of productivity. This is achieved by estimating the above four specifications for log productivity from U.S. data using the Kalman filtering technique. The maximum likelihood (ML) estimates and likelihood ratio statistics imply two notable results. First, the newly proposed productivity process that contains persistent trend growth shocks better describes the measured productivity in data than the other productivity specifications do. Second, persistent trend growth plays a crucial role in explaining the dynamics of productivity growth. This is because roughly 90% of fluctuations in aggregate productivity growth is attributable to variations in persistent trend growth.2)

2) To make a comparison with the work of Kaltenbrunner and Lochstoer (2010), I presume an inelastic labor supply, which implies that variations in productivity, as measured by the part of output that cannot be
The second, and major, contribution of this paper is to confirm the PIH prediction that justifies sizable long-run consumption risk in a production economy through persistent trend growth shocks that are directly estimated from the data. In order to match moments of interest, the previous literature has calibrated the variance of productivity at implausibly large values. Conversely, I incorporate the estimated parameters regarding the four productivity specifications into the production economy model and then compare the sizes of the LRR in consumption endogenously generated by each productivity process. The simulation study shows that the model characterized by the persistent trend growth generates a more substantial LRR in consumption than the model with the deterministic trend does. On average, the LRR under the stochastic trend and cycle is about 12 times larger than that under the deterministic trend and a cycle, and 8 times larger than that under the random walk with drift.

Due to the considerable size of the LRR in consumption, the persistent trend growth produces some salient features of macro quantities and asset prices, such as a moderate volatility of consumption growth relative to output growth that is consistent with the data, a high equity premium, a low risk-free rate, and the equity premium being a lead indicator of GDP growth. Furthermore, allowing for the persistent trend growth better explains a plausible volatility of the equity return and the risk-free rate than the deterministic trend.

This paper is mainly related to the literature on production-based asset pricing and long-run properties of productivity. Tallarini (2000) first combines the recursive utility with a general equilibrium production economy. Assuming no capital adjustment costs, the EIS being equal to unity, and a productivity shock following a random walk with drift, attributed to capital, include fluctuations in hours worked. I also present ML estimation results on productivity, as measured by the Solow residual that is consistent with an elastic labor supply. The results show that variations in persistent trend growth still explain 46% of variations in productivity growth with a first-order autocorrelation coefficient of 0.11.
Tallarini shows that asset prices are substantially affected by the RRA. However, in my work, the EIS is also shown to play an important role in determining the effect of the LRR in consumption on asset prices through two channels: the impatience channel and the precautionary saving channel. When the EIS is equal to one, the effects of the LRR through the two channels cancel, and thus the RRA that controls the effect of short-run consumption risk on asset prices matters only.

An important precursor is Kaltenbrunner and Lochstoer (2010) (KL hereafter), who point out that a persistent component of consumption growth can be endogenously created by the consumption smoothing motive in the production economy. In the process of drawing this conclusion, KL bring in two types of productivity processes: a deterministic trend along with a cycle and a random walk with drift. There, the productivity processes are calibrated so that implied variances are implausibly large. This paper adds to this work suggesting persistent trend growth shocks could generate a larger effect on long-run consumption risk than that found in KL.

My analysis relates directly to the literature on long-run structures of productivity news. Aguiar and Gopinath (2007) first show the benefits of taking into account an impact of a current, persistent shock to trend growth in an emerging market business cycle study. I first document the importance of persistent trend growth for U.S. in a production economy model. Differently from Aguiar and Gopinath, I assume that a persistent trend growth shock acts as a news shock. On the other hand, the literature on a news shock takes an i.i.d. news shock as given (e.g., Barsky and Sims 2011; Bretscher et al. 2018).

More recently, Croce (2014) presents evidence for the existence of a long-run productivity shock, which can be interpreted as a persistent trend growth shock in my framework, using two multivariate approaches—the price-dividend ratio and the risk-free rate—under the assumption of the productivity process that is the reduced form of my second specification. Croce indirectly suggests that LRR in productivity is...
the source of LRR in consumption by linking LRR in productivity to asset prices in a production economy. I contribute to this literature in two ways. First, I evaluate which of productivity processes fits U.S. data better and assess which one is more important in generating a sizable long-run consumption risk by directly connecting to LRR in consumption. Second, I use the rigorously estimated productivity parameter values as model inputs to avoid the concern that the results are reverse-engineered to match the macro moments.

II. Model

This section describes an extension of the growth model by introducing recursive preferences and capital adjustment costs, which is along the lines of Croce (2014) and KL. The new feature is that the data-generating process for productivity can include persistent trend growth shocks.

1. Preferences

The economy is assumed to be populated by identical households who have EZ preferences over a nondurable consumption good $C_t$:

$$V_t = \left( (1-\beta)C_t^{1-1/\psi} + \beta \left( E_t\left[ V_{t+1}^{1-\gamma} \right] \right)^{1-1/\psi} \right)^{1/(1-1/\psi)},$$

where $\beta$ is the subjective discount factor, $\psi$ the elasticity of intertemporal substitution (EIS), and $\gamma$ the coefficient of relative risk aversion (RRA). The representative household with EZ preferences values not only the consumption good, but also the certainty equivalent of continuation utility. It is well known that the EZ preferences allow the EIS to be decoupled from the RRA, and the relative sizes of the two determine the timing of
the uncertainty resolution. The representative household prefers early (late) resolution of uncertainty if $\gamma > 1/\psi$ ($\gamma < 1/\psi$).

The stochastic discount factor (SDF) is the intertemporal marginal rate of substitution of the representative household:

$$M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1/\psi} \left( \frac{V_{t+1}}{E_t \left[ V_{t+1}^{1-\gamma} \right]^{1/(1-\gamma)}} \right)^{1/\psi - \gamma}.$$  \hspace{1cm} (2)

If $\gamma \neq 1/\psi$, the SDF depends on the second term, future utility adjusted by its certainty equivalent, which captures news about future consumption growth. Some more insight into equation (2) can be obtained by looking at the log-linearized SDF.

**A close look at the stochastic discount factor**

The SDF is the key to generating asset returns. Investigating the result of a log-linear approximation of the household’s objective function yields some implications on the economy with EZ preferences, especially the form of the SDF. Note that the utility recursion, equation (1), implies

$$\left( \frac{V_t}{C_t} \right)^{1-1/\psi} = (1-\beta) + \beta E_t \left[ \frac{V_{t+1}^{1-\gamma}}{C_{t+1}^{1-\gamma}} \right]^{1-1/\psi}, \hspace{1cm} (3)$$

and the resulting log-linear approximation of equation (3) is as follows:

$$vc_t = \kappa(\epsilon) + \beta'(\epsilon)\epsilon e_t, \hspace{1cm} (4)$$

where $vc_t = \log \left( \frac{V_t}{C_t} \right)$ denotes the logarithm of the ratio of utility to

---

3) See Appendix B for a derivation of equation (4).
consumption; 
\[ ce_t = \frac{1}{1-\gamma} \log \left( E_t \left[ \frac{V_{t+1}}{C_{t+1}} \frac{C_{t+1}}{C_t} \right]^{1-\gamma} \right) \]
denotes the logarithm of the certainty equivalent of future utility scaled by current consumption; 
\( ce \) denotes the unconditional mean of \( ce_t \); and \( \beta'(ce) \) denotes an effective discount factor.

**Proposition 1.** The log-linear model has an effective discount factor \( \beta'(ce) \), which is defined by

\[
\beta'(ce) = \beta \frac{\exp((1-1/\psi)ce)}{1-\beta + \beta \exp((1-1/\psi)ce)}.
\]

The effective discount factor \( \beta'(ce) \) is increasing in \( ce \) if and only if \( \psi > 1 \).

That \( \beta'(ce) \) is increasing in \( ce \) if and only if \( \psi > 1 \) is easily verifiable, so I omit proof of this proposition. Proposition 1 shows that the effective discount factor \( \beta'(ce) \) is the original \( \beta \) multiplied by a term that is determined by \( ce \). Holding the RRA fixed and the EIS larger than unity, the effective discount factor implies that agents become more patient if they are in an economy featuring a large certainty equivalent on average. A question then arises: what components make up the certainty equivalent? Proposition 2 answers this.

**Proposition 2.** Under homoscedasticity and independence assumptions, the certainty equivalent is a decreasing function of the magnitude of both short-run risk (SRR) and long-run risk (LRR) in consumption growth:

\[
ce_t = \sum_{j=0}^{\infty} (\beta'(ce))^j \left\{ \kappa(ce) - 0.5(\gamma-1)\sigma(ce)^2 + E_t[\Delta c_{t+1+j}] \right\},
\]

where \( \Delta c_{t+1} = \log(C_{t+1}/C_t) \) is the logarithmic change in real, per capita consumption and
\[ \sigma(ce)^2 = \text{Var}_t \left[ \sum_{j=1}^{\infty} (\beta'(ce))^j (E_{t+1} - E_t) \Delta c_{t+1+j} \right] + \text{Var}_t \left[ (E_{t+1} - E_t) \Delta c_{t+1} \right] \]

See Appendix B for proof.

This result shows that the economy with dangers, measured by not only the amount of the SRR but also the amount of the LRR, would have low certainty equivalent if \( \gamma > 1 \). In other words, risk attitudes that agents in the economy may have are determined by not only the RRA but also the size of the LRR and SRR in consumption. With the certainty equivalent, one can now bind the SDF and news about current and future consumption growth together.

**Proposition 3.** The resulting log-linearized revisions in the SDF is a function of the revisions in realized (SRR) and future (LRR) consumption growth:

\[ (E_{t+1} - E_t)m_{t+1} = -\gamma (E_{t+1} - E_t) \Delta c_{t+1} - \left( \gamma - \frac{1}{\psi} \right) \sum_{j=1}^{\infty} (\beta'(ce))^j (E_{t+1} - E_t) \Delta c_{t+1+j}, \]

where \( m_{t+1} = \log(M_{t+1}) \).

See Appendix B for proof.

Proposition 3 is similar to the expression for revisions in the log SDF in Campbell (1999) and KL, but the difference comes from the way the discount factor is defined. While those works utilized the wealth to consumption ratio, I employ the certainty equivalent of continuation utility normalized by current consumption. The SDF measures growth in the desire for consumption. Long-run consumption risk does not contribute to the growth in the desire for consumption if \( \gamma = 1/\psi \), which coincides with the case of a time-separable power utility function. In contrast, when \( \gamma \neq 1/\psi \), long-run consumption news is an additional source of risk of a log SDF shock.
This feature has the considerable advantage in linking the prediction of the PIH to asset prices. Let’s imagine the situation where agents hear the bad news that their income will gradually decrease for the next five years, starting from the next quarter, due to a severe recession. The drop in the expectation of future income will lead to a persistent decline in consumption growth in the future if the agents prefer early resolution of uncertainty \((\gamma > 1/\psi)\). This hypothetical situation raises the growth in the desire for consumption, \(m_{t+1}\). Hence, the agents avoid risky assets that pay off badly in recessions and demand high risk premium for buying them. The agents also tend to save more than before, which gives rise to a low risk-free rate of return.

2. Technologies

The current consumption good is made using capital \(K_t\) and hours worked \(H_t\) through a Cobb-Douglass production function:

\[
Y_t = K_t^\alpha (A_t H_t)^{1-\alpha},
\]

where \(Y_t\) is output, \(A_t\) labor augmenting productivity, and \(\alpha \in (0, 1)\) a parameter that measures capital’s share of output. Following KL, hours worked are assumed to be one \((H_t = 1)\). That is, the representative household does not value leisure and thus fully supplies labor in this model economy.

The representative firm’s capital accumulation is subject to adjustment costs. The capital stock evolves as follows:

\[
K_{t+1} = (\Phi(I_t/K_t) + 1 - \delta)K_t,
\]

where \(I_t\) is investment and \(\delta\) is the depreciation rate. The resource constraint is given by
Following Jerman (1998), a concave function $\Phi(\cdot)$ captures convex capital adjustment costs:

$$
\Phi(I_t/K_t) = a_0 + \frac{a_1}{1-1/\xi} (I_t/K_t)^{1-1/\xi},
$$

where $\xi$ is the Tobin’s $q$ elasticity of investment-capital ratio. The higher $\xi$ is, the smaller the adjustment costs are. The parameters $a_0$ and $a_1$ are chosen such that there is no capital adjustment cost at the steady state; that is, $\Phi(I/K) = I/K$ and $\Phi'(I/K) = 1$.

### 3. Productivity process

In order to compare the effects of productivity specifications on macro variables and asset returns, I propose a generalized version of I(1) productivity process that is able to allow for four processes: a trend-stationary (TS) process with persistent trend growth, a difference-stationary (DS) process with persistent trend growth, a TS process with deterministic trend growth, and a DS process with i.i.d. trend growth. The level of log productivity $a_t$ is

$$
a_t = \tau_t + z_t,
$$

where $\tau_t$ represents a base trend and $z_t$ is a stochastic, possibly periodic or permanent, component. In what follows, the two decomposed components will be assumed to be independent from each other.

Without innovations, the non-stationary base trend component has a constant drift $\mu$, which captures the steady state growth rate of productivity, and a stochastic drift $\sigma_t$:
\[ \tau_t = \mu + s_{t-1} + \tau_{t-1}. \]  

\[ (6) \]

\( \mu + s_{t-1} \) can be interpreted as the slope of the base trend or the stochastic base trend growth, and its time-varying component follows an AR(1) process:

\[ s_t = \rho_s s_{t-1} + \epsilon_{s,t}, \]  

\[ (7) \]

where \( \epsilon_{s,t} \sim N(0, \sigma_s^2) \) for all \( t \), \( |\rho_s| < 1 \), and \( \sigma_s \geq 0 \). Thus, the growth rate of the base trend is stochastic if \( \sigma_s > 0 \). When \( \sigma_s = 0 \) and \( s_0 = 0 \), the base trend component collapses to a deterministic trend.

The other component \( z_t \) can be stationary (I(0)) or integrated of order 1 (I(1)). The stochastic process is modeled as

\[ z_t = \rho_z z_{t-1} + \epsilon_{z,t}, \]  

\[ (8) \]

where \( \epsilon_{z,t} \sim N(0, \sigma_z^2) \) for all \( t \), \( \sigma_z > 0 \), and \( |\rho_z| < 1 \) or \( \rho_z = 1 \). Again, the two shocks are assumed to be independent of each other; that is, \( E_t[\epsilon_{s,t}\epsilon_{z,t}] = 0 \).

Under the assumption that \( z_t \sim I(0) \) (i.e., \( |\rho_z| < 1 \)), equation (5) is a TS process because the stationary component \( z_t \) can be interpreted as a cycle that captures deviations from the trend. Hence, a shock to the cycle \( \epsilon_{z,t} \) only has a temporary impact on \( a_t \). On the other hand, if \( z_t \sim I(1) \) (i.e., \( \rho_z = 1 \)), \( z_t \) becomes an additional stochastic trend component, and thus (5) is of the DS form. This interpretation arises from the fact that as \( z_t \) is a pure random walk process without drift (i.e., \( z_t \) is the accumulation of current and past shocks), shocks have a permanent effect on \( a_t \). This, of course, implies that the effect of \( \epsilon_{z,t} \) on the trend growth is unpredictable.\(^4\)

\(^4\) Harvey (1985) and Clark (1987), among others, consider the stochastic trend growth as being integrated of
This specification for productivity nests four specifications as special cases: 1) a TS process with persistent trend growth, 2) a DS process with persistent trend growth, 3) a TS process with deterministic trend growth, and 4) a DS process with i.i.d. trend growth.

**Specification 1: TS process with persistent trend growth**

Throughout this paper, the benchmark specification is the productivity process featuring the base trend with persistent growth and the cycle. This specification is obtained by imposing the following restrictions on the parameters: $\sigma_s > 0$ and $|\rho_z| < 1$.

The benchmark process can be thought of as a TS process. To see why, it is useful to express $a_t$ in the following way:

$$a_t = \tau_0 + \mu t + (1 - \rho_s L)^{-1} \sum_{j=1}^{t-1} \epsilon_{s,j} + z_t,$$

where $s_0 = 0$ is assumed. The first two terms in (9) represent a deterministic trend, and the third term refers to a stochastic trend. Thus, the first three terms measure the overall trend ($\sigma_{\tau}$). As removing the overall trend leaves $z_t$, which follows a stationary and invertible AR(1) process, the benchmark process can be considered as a TS process. Additionally, it must be noted that $\epsilon_{s,t}$ in (7) acts as a news shock. It makes no instant impact on the level of log productivity. However, since agents can observe this shock, it serves as a signal of the next period’s log productivity.

This specification is in line with Aguiar and Gopinath (2007) who conceived the trend process as:

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order 2. However, Harvey (1985) reports that since the variance of a permanent shock to trend growth appears to be quite small, an I(1) stochastic trend is unlikely to mislead his conclusion.
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\[ \tau_t = s_{t-1} + \tau_{t-1} \]  
\[ s_t = (1 - \rho_s)\mu + \rho_s s_{t-1} + \epsilon_{s,t}. \]

The fundamental difference between specification 1 and the specification considered above can be expressed in terms of the conditional expectation of the overall trend growth, \( E_{t-1}[\Delta \sigma_t] \). The conditional expectation implied by specification 1 is \( \mu + s_{t-1} \), whereas equations (10) and (11) imply \( (1 - \rho_s)\mu + \rho_s s_{t-1} \). This difference causes the LRR in overall trend in specification 1 to be bigger than that in the Aguiar and Gopinath’s specification by \( \sigma_s^2 \).

**Specification 2: DS process with persistent trend growth**

Specification 2 is made by imposing \( \sigma_z > 0 \) and \( \rho_z = 1 \), and it has a form similar to (9):

\[ a_t = \tau_0 + \mu t + (1 - \rho_s L)^{-1} \sum_{j=1}^{t-1} \epsilon_{s,j} + \sum_{j=1}^{t} \epsilon_{z,j}. \]  
\[ (12) \]

Equation (12) shows that specification 2 has a deterministic trend and two stochastic trends in the sense that two different shocks, \( \epsilon_{s,t} \) and \( \epsilon_{z,t} \), have permanent effects on the level of log productivity. As in specification 1, the persistent growth rates come from the third component. The last trend term has white noise shocks with variance \( \sigma_z^2 \), which do not contribute to prediction of the productivity growth rates.

By first differencing, we obtain the following reduced form representation for the change in log productivity:

\[ \Delta a_t = \mu + s_{t-1} + \epsilon_{z,t} \]  
\[ (13) \]
\[ s_t = \rho_s s_{t-1} + \epsilon_{s,t}. \]  
\[ (14) \]
In (13), both $s_{t-1}$ and $\varepsilon_{z,t}$ are stationary and invertible ARMA processes. In this sense, specification 2 is a DS process. Equations (13) and (14) are the same as the specification proposed by Croce (2014) who defined $s_t$ as the long-run component in productivity growth and $\varepsilon_{z,t}$ as short-run growth risk.

**Specification 3: TS process with deterministic trend growth**

With $\sigma_s > 0$, specification 1 collapses to the following system:

$$ a_t = \tau_t + z_t, \quad \tau_t = \mu + \tau_{t-1}, \quad z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, $$

(15)

where $|\rho_z| < 1$. Equation (15) reduces to $a_t = \tau_0 + \mu t + z_t$ and $z_t = \rho_z z_{t-1} + \varepsilon_{z,t}$, which are of the conventional TS form. Specification 3 is a traditional setup in macro-asset pricing studies, and it is a popular approach to describing economic time series in the business cycle literature.

**Specification 4: DS process with i.i.d. trend growth**

The log productivity of specification 4 follows equation (15) with autoregressive coefficient $\rho_z = 1$. Specification 4 contains a deterministic trend and a stochastic trend whose growth is unpredictable. This specification can be also written as a random walk with drift, $a_t = \mu + a_{t-1} + \varepsilon_{z,t}$, which is of the simplest DS form. Its first difference is

$$ \Delta a_t = \mu + \varepsilon_{z,t}. $$

(16)

Tallarini (2000) and KL use this process to analyze the effect of permanent shocks on optimal consumption-saving behavior.5)

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5) KL examine the effect of a productivity shock’s persistence on the magnitude of the LRR in consumption by comparing specifications 3 and 4, and concluded that under the assumption of a preference for early resolution
Table 1. Summary of productivity process

<table>
<thead>
<tr>
<th>Specification</th>
<th>Deterministic trend</th>
<th>Stochastic trend with persistent growth</th>
<th>Stochastic trend with i.i.d growth</th>
<th>Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specification 1</td>
<td>$\tau_0 + \mu t$</td>
<td>$\sum_{j=1}^T s_j$</td>
<td>$\sum_{j=1}^T \epsilon_{z,j}$</td>
<td>$z_t$</td>
</tr>
<tr>
<td>Specification 2</td>
<td>$\tau_0 + \mu t$</td>
<td>$\sum_{j=1}^T s_j$</td>
<td>$\sum_{j=1}^T \epsilon_{z,j}$</td>
<td>$z_t$</td>
</tr>
<tr>
<td>Specification 3</td>
<td>$\tau_0 + \mu t$</td>
<td>$\sum_{j=1}^T s_j$</td>
<td>$\sum_{j=1}^T \epsilon_{z,j}$</td>
<td>$z_t$</td>
</tr>
<tr>
<td>Specification 4</td>
<td>$\tau_0 + \mu t$</td>
<td>$\sum_{j=1}^T \epsilon_{z,j}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Asset prices

With the above SDF, it is straightforward to calculate asset returns. First, the one-period risk-free rate is defined as $R_{k,t} = 1/E_t [M_{t+1}]$. I consider two more returns that are associated with two kinds of dividend streams: unlevered firm and levered consumption dividends.

The return on capital

In this production economy, the payouts (dividends) paid by the firm are given by $D_{k,t} = \alpha Y_t - I_t$. The return on capital is the return on the claim to the firm’s payouts, which is defined as:

$$R_{k,t+1} = \frac{D_{k,t+1} + q_{t+1}K_{t+2}}{q_tK_{t+1}},$$

where $q_t = [\Phi'(L_t/K_t)]^{-1}$ is the shadow price of one unit of newly installed capital, which is also interpreted as the marginal rate of of uncertainty, specification 4 strengthens the precautionary savings effect, and thus increases the LRR in consumption, compared to specification 3. This is presumably due to the different size of the long-run impact of a shock $\epsilon_{z,t}$ on the level of log productivity $\alpha$. Theoretically, specification 4 implies that $z_t$ increases at by that same amount in the long-run whereas specification 3 implies no long-run impact.
transformation between consumption and new capital, and 
\[ K_{i+2}/K_{i+1} = \Phi(I_{i+1}/K_{i+1}) + 1 - \delta. \] In equilibrium, the return on capital is required to satisfy the Euler condition:

\[ 1 = E_t[M_{t+1}R_{k,t+1}]. \]

The return on levered consumption

The return on capital may not be a good gauge of the equity market return. In reality, capital stocks that listed firms hold are only a small fraction of all capital stocks (see KL). In order to fit the data on the equity market return, I assume that the levered consumption claim is the same as the claim to dividend streams from public corporations, which is an accepted approach as found in the finance literature. BY support this point of view in the sense that they find that the correlation between consumption growth and dividend growth in the data is around 0.55. Following Abel (1999), BY, and Bretscher et al (2018), dividend growth in stock markets is given by:

\[ \frac{D_{m,t+1}}{D_{m,t}} = \exp((1 - \phi)\mu)\left(\frac{C_{t+1}}{C_t}\right)^\phi, \]

where \( \phi \) is the leverage parameter and \( \mu \) is the average growth rate of productivity. If \( \phi = 1 \), the claim to dividends is exactly the same as the claim to aggregate consumption. If \( \phi > 1 \), dividend growth becomes much more volatile than consumption growth.

The levered consumption return is defined as follows:

\[ R_{m,t+1} = \frac{P_{m,t+1} + D_{m,t+1}}{P_{m,t}}, \quad (17) \]

and it must satisfy the following recursive equation with respect to the
III. Estimation of Productivity Process

This section documents persistent trend growth shocks in U.S. data on quarterly productivity time series from 1955—2014 and provides estimates of the four specifications introduced in the previous section using the exact initial Kalman filter. The estimated parameters are used as inputs in the growth model. I begin with a brief discussion of productivity measure.

1. Productivity measure

As I assume inelastic labor supply in the model, I measure productivity as the part of per capita real output not explained by the amount of capital used in production. Real output is the sum of real consumption and investment as in the resource constraint. Data for U.S. real consumption (nondurables and services) and investment (nonresidential fixed investment) is taken from the National Income and Product Account (NIPA) dataset. All real variables are adjusted by total population, as measured by Current Population Survey (CPS) civilian non-institutional population aged 16 to 64. The capital stock series is constructed by the perpetual inventory method. Appendix E describes the data sources and the measurement of productivity in detail. In order to measure productivity, a calibration of a set of parameters ($\alpha$ and $\delta$) for the production technology is needed. A period in the model is one quarter, so it is sensible to set the parameters for the production technology to values commonly used in the business cycle literature. In particular, the

$$\frac{P_{m,t}}{D_{m,t}} = E_t \left[M_{t+1} \frac{D_{m,t+1}}{D_{m,t}} \left(\frac{P_{m,t+1}}{D_{m,t+1}} + 1\right)\right].$$
capital share $\alpha$ is $1/3$, and the depreciation rate $\delta$ is 0.02.

2. Parameter estimation

An identification issue often arises when decomposing an observable I(1) process into unobservable components (e.g., Nelson and Plosser 1982; Clark 1987). It should be noted that specifications 1 and 2 are always identified, irrespective of whether $z_t$ is transitory or permanent. While specification 2 is exactly identified, specification 1 is overidentified under the assumption that the correlation between innovations in the trend and cycle components is zero. This result is described in Appendix C. Specification 1 can allow for dependence between the trend and cycle components, but I assume that they are independent to facilitate comparison with specification 2.\(^6\)

I estimate the productivity process on U.S. quarterly data ranging from 1955:1 to 2014:4. The exact initial Kalman filter is applied to obtain quasi-maximum likelihood estimates of the parameters because initial state vectors are fully or partially diffuse across all specifications.\(^7\) Once productivity process is represented in a state-space form, the filter can be easily applied. After application of the filter, we can infer unobservable components.

Specification 1 can be expressed as the following state-space form:\(^8\)

\[
a_t = Z\alpha_t, \\
\alpha_t = \tilde{\mu} T \alpha_{t-1} + R \eta_t, \quad \eta_t \sim N(0, Q),
\]

---

6) In Appendix H, I relax the restriction of the zero correlation between trend and cycle shocks and estimate it by maximum likelihood. The estimate is strongly positive, which is in sharp contrast to the Beveridge and Nelson (1981) decomposition. The likelihood ratio test statistics for the zero correlation restriction is 2.115 with a bootstrap $p$-value of 0.097. However, the unconstrained model and the zero correlation model yield the similar result in terms of fluctuations in estimated trend growth.

7) See Durbin and Koopman (2012) for a comprehensive review of the exact initial Kalman filter.

8) The state space form of specification 2 is explained in Appendix F.
for $t = 1, \cdots, T$, where $Z = [1 1 0]$; $\alpha_t = [\tau_t z_t s_t]^T$; $\tilde{\mu} = [\mu 0 0]^T$; and $\eta_t = [\epsilon_{z,t} \epsilon_{s,t}]^T$. Elements of the matrices $T$, $R$, and $Q$ are constant such that

$$T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & \rho_z & 0 \\ 0 & 0 & \rho_s \end{bmatrix}, \quad R = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{and} \quad Q = \begin{bmatrix} \sigma_z^2 & 0 \\ 0 & \sigma_s^2 \end{bmatrix}. $$

Since $\tau_t$, the element of the state vector $\alpha_t$, is non-stationary, the initial state vector $\alpha_0$ is partially diffuse. Following Durbin and Koopman (2012), I specify the initial state vector $\alpha_0$ as

$$\alpha_0 = B_0 \tau_0 + R_0 \eta_0, \quad \eta_0 \sim N(0, Q_0),$$

where $\tau_0 \sim N(0, \kappa)$ is a scalar to be diffuse as $\kappa \to \infty$; $\kappa$ is the unconditional variance of the non-stationary initial state variable $\tau_0$; and $Q_0$ is the unconditional covariance matrix of the stationary initial state variables $z_0$ and $s_0$. The matrices $B_0$, $R_0$, and $Q_0$ are given by

$$B_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad R_0 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{and} \quad Q_0 = \begin{bmatrix} \sigma_z^2(1-\rho_z^2)^{-1} & 0 \\ 0 & \sigma_s^2(1-\rho_s^2)^{-1} \end{bmatrix}. $$

This specification leads to $\alpha_0 \sim N(0, P_0)$, $P_0 = \kappa P_\infty + P_\ast$, where $P_\infty = B_0 B_0^T$ and $P_\ast = R_0 Q_0 R_0^T$. When $P_0$ depends on $\kappa \to \infty$, the standard Kalman filter that is initiated by a large, arbitrary value for $\kappa$ can lead to large rounding errors. On the other hand, the exact initial Kalman filter seeks to avoid the possible rounding errors by expanding the inverse of variance of the prediction error as power series in $\kappa^{-1}$ and then by letting $\kappa \to \infty$ to obtain dominant terms.

Let $P_{t|t-1}$ be the covariance matrix of $\alpha_t$ conditional on information up to $t-1$ and $F_t$ be the variance of the prediction error at $t$. Then
the unconditional covariance matrix of the initial state vector
\[ P_0 = \kappa P_\infty + P_\sigma \]
implies
\[ P_{t|t-1} = \kappa P_{\infty,t|t-1} + P_{\sigma,t|t-1} + O(\kappa^{-1}) \]
and
\[ F_t = \kappa F_{\infty,t} + F_{\sigma,t} + O(\kappa^{-1}). \]
Koopman (1997) shows that
\[ P_{t|t-1} = P_{\sigma,t|t-1} \]
and \[ F_t = F_{\sigma,t} \]
for \( t = d+1, \ldots, T \) using the fact that the rank of \( P_{\infty,t|t-1} \) is generally equal to the rank of \( P_{\infty,t|t-1} \) minus the rank of \( F_{\infty,t} \). We can therefore implement the standard Kalman filter after the time period \( d \). For \( t \leq d \), \( F_{\infty,t} \) is a nonnegative value due to \( P_{\infty,t|t-1} \neq 0 \) by definition of \( d \). Consider the case where \( F_{\infty,t} \) is positive.\(^9\)

Expanding the inverse of variance of the prediction error \( F_{t-1} \) as power series in \( \kappa^{-1} \) yields
\[ \lim_{\kappa \to \infty} \left( -\log F_t + \log \kappa \right) = -\log F_{\infty,t} \quad \text{and} \quad \lim_{\kappa \to \infty} v_t^T F_t^{-1} v_t = 0, \tag{18} \]
where \( v_t = a_t - E_{t-1}[a_t] \) is the prediction error.

I now evaluate the diffuse log-likelihood function for \( \{a_t\}_{t=1}^T \) in specification 1, which is invariant to the diffuse element \( \tau_0 \). The diffuse log-likelihood is defined as
\[ \log L_d(\{a_t\}_{t=1}^T) = \lim_{\kappa \to \infty} \left( \log L(\{a_t\}_{t=1}^T) + 0.5 \log \kappa \right), \tag{19} \]
where
\[ \log L(\{a_t\}_{t=1}^T) = -0.5 T \log 2\pi - 0.5 \sum_{t=1}^{T} \log F_t - 0.5 \sum_{t=1}^{T} v_t^T F_t^{-1} v_t. \] Putting (18) into (19), we can obtain the following diffuse log-likelihood:
\[ \log L_d(\{a_t\}_{t=1}^T) = -0.5 \left( T \log 2\pi + \sum_{t=1}^{d} \log F_{\infty,t} \right) - 0.5 \sum_{t=d+1}^{T} \left( \log F_t + v_t^T F_t^{-1} v_t \right). \tag{20} \]

\(^9\) Since the case \( F_{\infty,t} = 0 \) is not commonly occurring in practice, I consider the case where \( F_{\infty,t} \) is positive. For the case \( F_{\infty,t} = 0 \), see Durbin and Koopman (2012).
I estimate the parameters by maximizing the diffuse log-likelihood function (20). Numerical maximization of the diffuse log-likelihood function is conducted by the BFGS (Broyden-Fletcher-Goldfarb-Shanno) method. In Appendix G, I discuss log-likelihood evaluation via the exact initial Kalman filter and the standard Kalman filter.

3. Estimation results

Table 2 reports the parameter estimates for the four productivity specifications along with their standard errors. In order to compare goodness of fit among the four specifications, Akaike’s information criterion (AIC) and Bayesian information criterion (BIC) are presented alongside a likelihood ratio (LR) test. Since specifications 2, 3, and 4 are nested in the proposed process (5)-(8), and they are fitted using ML estimation, I conduct the likelihood ratio (LR) test of the null hypothesis that some parameters of each specification satisfy the imposed restrictions. As the sample size is finite \((T = 240)\), I employ a bootstrap method to estimate the sampling distribution of the likelihood ratio (LR) test statistic by replicating 10,000 samples with the same number of observation as the actual sample. The following conclusions can be drawn from the results.

First, when productivity has a deterministic time trend only, cycle shocks are unsurprisingly highly persistent. In specification 3 (column 3 of Table 2), the ML estimate of the cycle persistence is large—around 0.988, which is close to unity as in specification 4. Note that since I estimate the four specifications using an identical productivity series, the estimated parameters from all specifications must induce the same estimated variance of log productivity growth. Thus, the estimated standard deviations of \(\epsilon_{z,t}\) should be similar in specifications 3 and 4. Indeed, they are respectively 1.266% and 1.270%^{11}.

---

10) As specifications 1, 2, and 4 include diffuse initialization, information criteria must penalize not only the number of parameters, but also the number of nonstationary elements (Durbin and Koopman 2012).

11) These values are far smaller than the calibrated value of around 4.100% in KL, who calibrate the volatility
Table 2. Parameter Estimates

<table>
<thead>
<tr>
<th>Specification #</th>
<th>Spec. 1</th>
<th>Spec. 2</th>
<th>Spec. 3</th>
<th>Spec. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_i$ type :</td>
<td>Persistent trend growth</td>
<td>Deterministic trend growth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z_i$ type :</td>
<td>Cycle</td>
<td>Random walk</td>
<td>Cycle</td>
<td>Random walk</td>
</tr>
<tr>
<td>Specification #</td>
<td>Spec. 1</td>
<td>Spec. 2</td>
<td>Spec. 3</td>
<td>Spec. 4</td>
</tr>
<tr>
<td>Parameter</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>0.693 (0.066)</td>
<td>0.709 (0.069)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>$-0.054$ (0.407)</td>
<td>1.000</td>
<td>0.998 (0.013)</td>
<td>1.000</td>
</tr>
<tr>
<td>$\sigma_s \times 100$</td>
<td>0.875 (0.084)</td>
<td>0.816 (0.101)</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$\sigma_z \times 100$</td>
<td>0.263 (0.115)</td>
<td>0.531 (0.109)</td>
<td>1.266 (0.058)</td>
<td>1.270 (0.058)</td>
</tr>
</tbody>
</table>

Goodness of fit

<table>
<thead>
<tr>
<th>Goodness of fit</th>
<th>Spec. 1</th>
<th>Spec. 2</th>
<th>Spec. 3</th>
<th>Spec. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Likelihood</td>
<td>755,067</td>
<td>754,041</td>
<td>703,853</td>
<td>703,135</td>
</tr>
<tr>
<td>AIC</td>
<td>$-1500,134$</td>
<td>$-1498,082$</td>
<td>$-1403,706$</td>
<td>$-1402,270$</td>
</tr>
<tr>
<td>BIC</td>
<td>$-1482,731$</td>
<td>$-1480,679$</td>
<td>$-1396,745$</td>
<td>$-1395,309$</td>
</tr>
</tbody>
</table>

Likelihood Ratio Test

| LR | 2.052 | 102.428 | 103.864 |
| $p$-value | 0.172 | 0.000 | 0.000 |

Notes: This table reports parameter estimates of the four productivity processes for log productivity measurement $\alpha_t$, from 1955:1-2014:4. Standard errors are reported in parentheses. Log productivity $\alpha_t$ is assumed to consist of $\tau_i$ and $z_i$. I set $\mu$ as the sample mean of productivity growth. The exact initial Kalman filter is applied to obtain quasi-maximum likelihood estimates. The maximized log-likelihood values are reported together with the AIC and BIC. Following Durbin and Koopman (2012), specifications with more non-stationary state variables are penalized, leading to a larger value of the AIC and BIC. A bootstrap is used to estimate the sampling distribution of the likelihood ratio (LR) test statistic. I first obtain the shocks from the estimated specifications 2, 3, and 4. Note that the shocks in specification 2 are obtained from the smoothed state vector of the estimated specification 2. I then construct the re-sampled 10,000 data sets of the same size as the actual data for each specification by recursively simulating productivity series using the shocks sampled with replacement. For each bootstrap sample, I compute the LR test statistics. The $p$-value of the test is obtained as the proportion of 10,000 statistic replications greater than the sample value of the statistic.

Second, the cycle process under the persistent stochastic trend growth is different than would be expected. In specification 1 (column 1 of Table 2), the coefficient of the cycle persistence is not statistically significant with a $p$-value of 0.894, which implies that the cycle can be interpreted as an i.i.d. process. Although specification 1 has $\rho_s = 0.693$, close to...
\( \rho_s = 0.709 \) in specification 2, the two specifications induce different processes for \( z_t \): a white noise and a pure random walk, respectively. Under the assumption of independence between the underlying components, it seems that \( z_t \) does not have a great effect on the persistence of trend growth in specification 2.

Third, the AIC and BIC indicate that the specifications including the persistent stochastic trend growth fit the data better, and that the best fitting specification is the one with the transitory cycle (specification 1). The LR statistics and their p-values lead to the same conclusion as the information criteria do. The statistics provide evidence against the deterministic trend and cycle (specification 3) and the random walk with drift (specification 4) in favor of specification 1. Yet, we cannot reject specification 2 in favor of specification 1 at the 10% significance level. The information criteria and LR test lead to somewhat conflicting conclusions about whether specification 1 is better than 2. However, I use specification 1 as the benchmark throughout the paper for two reasons. First, the time-honored formulation of macroeconomic time-series features stationary deviations around a trend. Second, the p-value (0.172) in the LR test may be admissible in practical analysis. Likewise, few distinctions are made between specifications 3 and 4, in terms of the goodness of fit. The information criteria find no significant differences between two. Therefore, if the true data generating process is markedly different from specifications 3 and 4, a restriction on the parameter of the cycle component may not be critical.

Before making use of estimated parameters as inputs in the models, I re-estimate the parameters of specification 1, fixing the cycle persistence to zero. Since the original estimate of \( \rho_z = -0.054 \) was near zero, imposing the restriction should make little impact on \( \sigma_z \). This expectation is confirmed in Table 3 as the estimated \( \sigma_z \) of 0.276% is close to the corresponding value of 0.263% in Table 2.
Table 3. Productivity parameter values as model inputs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Spec. 1</th>
<th>Spec. 2</th>
<th>Spec. 3</th>
<th>Spec. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_s$</td>
<td>0.696</td>
<td>0.709</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.000</td>
<td>1.000</td>
<td>0.998</td>
<td>1.000</td>
</tr>
<tr>
<td>$\sigma_s \times 100$</td>
<td>0.870</td>
<td>0.816</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.074)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_z \times 100$</td>
<td>0.276</td>
<td>0.531</td>
<td>1.266</td>
<td>1.270</td>
</tr>
<tr>
<td></td>
<td>(0.057)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table provides the parameter values of each productivity process, which are used as model inputs in the simulation study. The estimates in specification 1 are obtained by maximizing the likelihood function after fixing $\rho_z$ to zero whereas the parameter values in specification 2, 3, and 4 are the same as reported in Table 2. Standard errors are reported in parentheses. The sample period is the first quarter of 1955 to the fourth quarter of 2014 with a total of 240 quarterly observations.

IV. Simulation Study

This section explores the importance of persistent trend growth shocks by comparing quantities and prices under the estimated four specifications. To discuss the quantitative implications, I numerically approximate the model using the envelope condition method proposed by Maliar and Maliar (2013).\(^1\)

Table 4. Quarterly model calibration

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>$\beta$</td>
<td>0.9979</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma$</td>
<td>10,000</td>
</tr>
<tr>
<td>Elasticity of substitution</td>
<td>$\psi$</td>
<td>1.6590</td>
</tr>
<tr>
<td>Capital share</td>
<td>$\alpha$</td>
<td>0.3333</td>
</tr>
<tr>
<td>Adjustment cost</td>
<td>$\xi$</td>
<td>28.195</td>
</tr>
<tr>
<td>Depreciation</td>
<td>$\delta$</td>
<td>0.0200</td>
</tr>
<tr>
<td>Mean productivity growth rate (%)</td>
<td>$\mu$</td>
<td>0.3676</td>
</tr>
<tr>
<td>Leverage factor</td>
<td>$\phi$</td>
<td>4.0000</td>
</tr>
</tbody>
</table>

\(^1\) See Appendix I for details.
1. Calibration

Table 4 reports calibrated parameters. I calibrate the capital share \( \alpha = 1/3 \) and the depreciation rate \( \delta = 0.02 \) using standard values from the business cycle literature. The average quarterly growth rate of productivity \( \mu \) is 0.367%, obtained from a direct measurement of productivity over the period 1955—2014. The EIS \( \psi \), time discount factor \( \beta \), and adjustment cost parameter \( \xi \) are set to match the relative volatilities of consumption and investment growth rates, and the first moment of the risk-free rates.\(^{13}\) The risk aversion parameter \( \gamma \) and the leverage factor \( \phi \) are respectively set to 10 and 4 across the models to match the first moment of the equity risk premium.

2. Macro quantities and the capital return

The capital return, consumption, and investment are simultaneously determined in the general equilibrium model while the risk-free rate and the return on equity are computed based on the SDF that is obtained from the model. Hence, I first explore the implications of the model on macroeconomic quantities and the capital return together under the four specifications for productivity process.

2.1 Moments of quantities

Table 5 reports simulated moments of macro variables and the capital return, using the estimated parameters in Table 3, alongside data moments. I begin by comparing the simulated moments between specification 3, in which the estimate of the cycle persistence is 0.988, and specification 4, where productivity shocks are permanent. Apart from

\(^{13}\) There has been debate about plausible values of the EIS. Recently, Chen et al. (2013) provide empirical evidence that the EIS is above unity by employing a semiparametric technique that can fully utilize nonlinear information about the recursive utility.
the relative volatility of consumption and investment growth, the models generate largely similar results between specifications 3 and 4; we do observe that the volatility of consumption growth relative to output growth is higher in specification 4. The reason for this difference may be explained by the PIH.

When agents observe a permanent and positive productivity shock, their expectations of increased future income lead to a boost in current consumption, according to the PIH. Panels E and F of Figure 2 plot the impulse response of the consumption growth rates to a positive 1% productivity shock. Agents in the economy experiencing permanent productivity shocks are 0.046% more likely to increase consumption than those encountering transitory shocks. The relative volatility of consumption and investment growth confirms the well-known fact that under the assumption of the deterministic base-trend, the optimal consumption-savings choice hinges on whether productivity shocks are permanent or transitory, even though the transitory shocks are highly persistent.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Data</th>
<th>Spec. 1</th>
<th>Spec. 2</th>
<th>Spec. 3</th>
<th>Spec. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Macroeconomic Quantities</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(\Delta y)(%)$</td>
<td>0.842</td>
<td>0.859</td>
<td>0.863</td>
<td>0.841</td>
<td>0.847</td>
</tr>
<tr>
<td>$\sigma(\Delta c)/\sigma(\Delta y)$</td>
<td>0.651</td>
<td>0.651</td>
<td>0.661</td>
<td>0.402</td>
<td>0.548</td>
</tr>
<tr>
<td>$\sigma(\Delta i)/\sigma(\Delta y)$</td>
<td>2.562</td>
<td>0.563</td>
<td>2.480</td>
<td>2.282</td>
<td>1.945</td>
</tr>
<tr>
<td>$AC_1(\Delta y)$</td>
<td>0.584</td>
<td>0.613</td>
<td>0.617</td>
<td>0.028</td>
<td>0.007</td>
</tr>
<tr>
<td>$AC_1(\Delta c)$</td>
<td>0.268</td>
<td>0.436</td>
<td>0.416</td>
<td>0.156</td>
<td>0.089</td>
</tr>
<tr>
<td>$AC_1(\Delta i)$</td>
<td>0.580</td>
<td>0.235</td>
<td>0.277</td>
<td>0.007</td>
<td>-0.019</td>
</tr>
<tr>
<td>$\rho(\Delta c, \Delta y)$</td>
<td>0.752</td>
<td>0.425</td>
<td>0.488</td>
<td>0.970</td>
<td>0.987</td>
</tr>
<tr>
<td>$\rho(\Delta i, \Delta y)$</td>
<td>0.891</td>
<td>0.906</td>
<td>0.903</td>
<td>0.996</td>
<td>0.996</td>
</tr>
<tr>
<td>$\rho(\Delta c, \Delta i)$</td>
<td>0.399</td>
<td>0.006</td>
<td>0.070</td>
<td>0.944</td>
<td>0.968</td>
</tr>
<tr>
<td><strong>Capital Returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(r_t)(%)$</td>
<td>0.216</td>
<td>0.217</td>
<td>0.431</td>
<td>0.383</td>
<td></td>
</tr>
<tr>
<td>$\sigma(r_t)(%)$</td>
<td>0.186</td>
<td>0.185</td>
<td>0.103</td>
<td>0.105</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The table reports statistics of the macro-quantities and the capital return based on simulated data from each model as well as from historical U.S. data; the data span the period from the first quarter of 1955 to the fourth quarter of 2014. $AC_1(\cdot)$ and $\rho(\cdot, \cdot)$ denote first-order autocorrelation and cross-correlation, respectively.
I now turn to a comparison between specifications 1 and 2. Recall that specification 1 considers a white noise cycle process whereas specification 2 features a pure random walk as $z_t$. As the PIH predicts, the model under specification 2 generates a higher relative volatility of consumption growth than the model with specification 1. When it comes to simulated moments other than the relative volatility, the models yield almost indistinguishable results between these two specifications.

However, it should be noted that the degree to which the persistence of $z_t$ determines consumption dynamics is not significant. A more decisive difference seems to be made by the base trend process. Compared to the deterministic base trend, the stochastic trend with persistent growth, generates more volatile consumption. I discuss the reason for this in detail in Section 4.3.

The data indicates that the autocorrelations of output, consumption, and investment growth are around 0.584, 0.268, and 0.580, respectively. Specifications 3 and 4 do much less well with the autocorrelations, generating autocorrelations that are too low relative to the data.\(^\text{14}\) In particular, they generate near-zero autocorrelations for investment growth. For specifications 1 and 2, the autocorrelations of output growth obtained from model simulations are more consistent with the data, although the predicted autocorrelations of consumption growth are higher than in the data.

With regard to contemporaneous correlations with output growth, all specifications capture the fact that the growth rates of consumption and investment are quite procyclical. Consumption growth is highly procyclical in specifications 3 and 4, even more so than in the data. On the other hand, while specifications 1 and 2 generate correlations of investment growth with output growth that are in line with the data value of 0.89, the correlations between consumption and output growth are respectively

\(^{14}\)Given that the growth rate of productivity is \textit{i.i.d.} in specification 4 and close to \textit{i.i.d.} in specification 3, it is reasonable that the autocorrelations are around zero.
0.425 and 0.488, smaller than the observed correlation of 0.752.

One more dimension that the model is able to explore is the contemporaneous correlation between consumption and investment growth. Table 5 shows that their correlation is moderate (0.399) in the data. When \( z_t \) is a single source of uncertainty, consumption and investment are highly correlated. This is clearly at odds with the data. However, independent stochastic trend growth allows them to be disentangled. In sharp contrast to the cases of deterministic base-trend, the models with persistent trend growth generate lower correlations. Overall, the models with specifications 1 and 2 tend to generate moderately positive correlations between consumption and investment growth, lower than the estimates in the data.

**Figure 2. Impulse responses of macro variables**

Panel A: Predictable trend shock \((\psi_1)\) in spec. 1

Panel B: Cycle shock \((\psi_2)\) in spec. 1

Panel C: Predictable trend shock \((\psi_2)\) in spec. 2

Panel D: Unpredictable trend shock \((\psi_2)\) in spec. 2

Panel E: Cycle shock \((\psi_2)\) in spec. 3

Panel F: Unpredictable trend shock \((\psi_2)\) in spec. 4

Notes: Impulse response of the growth rates of consumption, investment, and output in response to a positive one-standard deviation shock. The shock materializes at time 0.
2.2 Moments of the capital return

The bottom panel of Table 5 shows the first and second moments of the return to a capital claim. The first moment of the capital return is much lower under the stochastic trend with persistent growth, in productivity (specifications 1 and 2) than under the deterministic trend and a cycle (specification 3) and even the random walk with drift (specification 4). Plus, when stochastic trend growth is persistent, the capital return is more volatile than in both specifications 3 and 4. These results are intuitive in light of the PIH. If agents face productivity with stochastic trend, the persistent trend shocks to productivity may make them more concerned about future income risk, thereby causing them to save more. A noteworthy feature is that both specifications 1 and 2 generate similar first and second moments of the capital return. This may seem to go against the PIH because specification 2 specifies $\epsilon_{z,t}$ as a permanent shock whereas specification 1 features a transitory shock $\epsilon_{z,t}$. I examine this issue in Section 4.3.

3. Understanding consumption volatility differences

We have seen differences in the relative volatility of consumption growth across specifications. The significant differences appear to arise from whether agents face persistent trend growth in productivity. The PIH predicts that trend growth shocks to productivity may pose a far greater risk of future income than in the case of deterministic trend growth, thereby causing the expected future consumption growth to be more volatile. In this subsection, I investigate this mechanism.

15) Two things should be noted. First, since I assume inelastic labor, aggregate income risk comes purely from productivity risk. Second, capital is a fairly safe asset and the sole tool of saving for agents in this economy.
3.1 Why do differences in consumption volatility arise?

In order to verify the PIH prediction, I quantify the source of consumption growth volatility by decomposing consumption growth into the conditional expectation of consumption growth and innovation:

$$\Delta c_{t+1} = E_t[\Delta c_{t+1}] + \epsilon_{t+1}. $$

This expression provides a useful perspective on the LRR and SRR in consumption. One source of variation in consumption growth arises from fluctuations in the conditional expectation of consumption growth; that is, the LRR in consumption. Variability also comes from fluctuations in revisions in realized consumption growth: the SRR in consumption.

### Table 6. Decomposition of consumption growth

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Spec. 1</th>
<th>Spec. 2</th>
<th>Spec. 3</th>
<th>Spec. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2(\Delta c_{t+1})$ (%)</td>
<td>(0.560%)2</td>
<td>(0.571%)2</td>
<td>(0.338%)2</td>
<td>(0.400%)2</td>
</tr>
<tr>
<td>$\sigma^2(E_t[\Delta c_{t+1}])$</td>
<td>0.311</td>
<td>0.289</td>
<td>0.073</td>
<td>0.075</td>
</tr>
<tr>
<td>$\sigma^2(E_t[\Delta c_{t+1}])$</td>
<td>(0.312%)2</td>
<td>(0.307%)2</td>
<td>(0.091%)2</td>
<td>(0.110%)2</td>
</tr>
<tr>
<td>$AC_G(\Delta c_{t+1})$</td>
<td>0.436</td>
<td>0.416</td>
<td>0.156</td>
<td>0.162</td>
</tr>
<tr>
<td>$AC_G(E_t[\Delta c_{t+1}])$</td>
<td>0.927</td>
<td>0.938</td>
<td>0.954</td>
<td>0.948</td>
</tr>
<tr>
<td>$E(\epsilon_{t+1})$</td>
<td>0.185</td>
<td>0.196</td>
<td>2.987</td>
<td>2.079</td>
</tr>
</tbody>
</table>

Notes: The table reports variances and correlations of consumption growth and expected consumption growth, together with expectation of the log of the certainty equivalent of future utility adjusted by current consumption for simulated data from the model.

The second and third rows of Table 6 exhibit that the stochastic trend with persistent growth plays a very important role in generating a sizable LRR in consumption, which confirms the prediction of the PIH. It is also consistent with the result on the return on capital in light of the fact that the capital return reflects a saving motive that is influenced by the LRR. What matters, for the size of the LRR, is the base trend $\tau_t$ type (that is, whether trend growth is stochastically persistent or deterministic) rather
than the type of $z_t$ (that is, whether it is permanent or transitory.)

The fourth and fifth rows of Table 6 demonstrate that the sizable autocorrelation of consumption growth is attributed to the LRR in consumption. Although the deterministic trend and a cycle (specification 3) and the random walk with drift (specification 4) generate a fairly high autocorrelation of conditional expectation of consumption growth, the autocorrelation of consumption growth is below 0.200. On the other hand, while the stochastic base trend induces a lower autocorrelation of the conditional expectation of consumption growth than the deterministic base trend does, it induces a much higher autocorrelation of consumption growth.

At this point, a question arises: why does specification 1 generate a larger LRR in consumption than specification 2 does? This phenomenon may be counterintuitive to given the PIH because, while specification 2 includes a permanent component, specification 1 contains an i.i.d. process as $z_t$.

### 3.2 Importance of trend growth shocks

The answer to this question is given by the relative volatility of trend growth ($\Delta \tau$), which is persistent, to productivity growth ($\Delta a$), which captures the importance of trend growth shocks in explaining variation in productivity growth. To calculate the relative volatility, I divide the variance of changes in base trend by the variance of productivity growth.

#### Table 7. Relative volatility of persistent trend growth

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Persistent trend growth</th>
<th>Cycle</th>
<th>Random walk</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2(\Delta \tau_t)/\sigma^2(\Delta a_t)$</td>
<td>0.910</td>
<td>0.826</td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table reports the importance of persistent trend growth relative to productivity growth in both specifications 1 and 2. The denominator is directly obtained from the data. The numerator is obtained by $\sigma^2/\sigma^2_a$ using the estimates in Table 3. The sample period is the first quarter of 1955 to the fourth quarter of 2014.
As Table 7 shows, in the economy characterized by the stochastic trend with persistent growth and the transitory cycle, the stochastic trend component is much more important than it is in the economy characterized by the stochastic trend with persistent growth and the pure random walk component. This result suggests that when trend growth shocks are persistent ($\rho_s = 0.693$), the cycle plays a marginal role in generating the LRR in consumption.

4. The risk–free rate and the equity premium

In this subsection, I explore the implication of LRR in consumption on two asset returns: the risk-free rate and the return on the levered consumption claim. Table 8 demonstrates the first two moments of the risk-free rate and the risk premium of the levered consumption claim, along with their interaction with macro variables.

Table 8. Moments of asset returns and interaction with macro variables

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Data</th>
<th>Spec. 1</th>
<th>Spec. 2</th>
<th>Spec. 3</th>
<th>Spec. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(r_f)$ (%)</td>
<td>0.226</td>
<td>0.226</td>
<td>0.225</td>
<td>0.430</td>
<td>0.375</td>
</tr>
<tr>
<td>$\sigma(r_f)$ (%)</td>
<td>0.688</td>
<td>0.186</td>
<td>0.183</td>
<td>0.055</td>
<td>0.066</td>
</tr>
<tr>
<td>$E(r_m - r_f)$ (%)</td>
<td>1.724</td>
<td>1.608</td>
<td>1.603</td>
<td>0.002</td>
<td>0.413</td>
</tr>
<tr>
<td>$\sigma(r_m - r_f)$ (%)</td>
<td>8.373</td>
<td>7.184</td>
<td>7.191</td>
<td>0.426</td>
<td>4.147</td>
</tr>
<tr>
<td>$SR_{m}$</td>
<td>0.206</td>
<td>0.222</td>
<td>0.221</td>
<td>0.005</td>
<td>0.100</td>
</tr>
<tr>
<td>$\rho(r_m - r_f, \Delta y)$</td>
<td>0.092</td>
<td>-0.001</td>
<td>0.084</td>
<td>0.993</td>
<td>0.998</td>
</tr>
<tr>
<td>$\rho(r_m - r_f, \Delta i)$</td>
<td>-0.042</td>
<td>-0.383</td>
<td>-0.309</td>
<td>0.989</td>
<td>0.999</td>
</tr>
</tbody>
</table>

Notes: The table reports statistics of the risk-free rate and the equity premium. It presents results based on historical U.S. data from 1955:1 to 2014:4 and simulated data from each specification. $SR$ and $\rho(\cdot, \cdot)$ denote the Sharpe ratio and first-order cross-correlation, respectively.
4.1 The risk-free rate

The first row shows that, given persistent trend growth in productivity, the first moment of the risk-free rate generated by the model is consistent with the data. Meanwhile, productivity with deterministic trend and cycle generates an estimate that is almost twice as much as the observed moment. The model with the unpredictably growing stochastic trend (the random walk with drift) generates a mean risk-free rate of 0.375%, which is between what the stochastic base trend (specifications 1 and 2) and the deterministic trend (specification 3) yield. These results can be explained in the framework of the precautionary saving motive. To see this, consider the following risk-free rate, which is obtained from log-linear approximation:

\[ r_{f,t} = \frac{-\log \beta + 0.5 \sigma_1^2 + (1/\psi) E_t [\Delta c_{t+1}] - 0.5 \sigma_2^2}{\text{time preference}}. \]  

(21)

where \( \sigma_1^2 = (1/\psi - \gamma)(1 - \gamma)(LRR + SRR) \) and \( \sigma_2^2 = (1/\psi - \gamma)^2 LRR + \gamma^2 SRR. \)

In equation (21), the first two terms \((-\log \beta + 0.5 \sigma_1^2)\) comprise the time preference (or impatience) component, and the fourth term \((-0.5 \sigma_2^2)\) arises due to a demand for precautionary saving. This expression implies that the LRR and SRR in consumption play different roles through two channels. The impatience term is the first channel through which the LRR and SRR will lead to a higher or lower interest rate, which depends on the values of the EIS and the RRA. With the model parameter values \( (\psi = 1.659 \text{ and } \gamma = 10) \), large LRR and SRR induce a low certainty equivalent (ce), thereby making people more impatient. The risk-free rate therefore would increase through the impatience channel.

The second channel is precautionary saving through which the LRR and SRR decrease the interest rate. In the face of a higher aggregate income risk, agents would like to hedge against a more unfavorable and sizable consumption risk by saving more. Hence, in equilibrium, the
risk-free rate must fall in response to the increase in demand for saving.

The total effect of consumption risk on the interest rate through these two channels depends on $\psi$ and $\gamma$. If the agents prefer early resolution of uncertainty $(1/\psi - \gamma < 0)$, the SRR always exerts a stronger precautionary savings effect than impatience effect, whereas for the LRR the precautionary channel dominates the impatience channel if and only if $\psi > 1$.\textsuperscript{16} Taking our parameters as given, both the SRR and LRR always lower the risk-free rate, and their magnitudes help to determine the level of the risk-free rate.

I now put forth an intuitive explanation about how the different productivity processes are connected to the risk-free rate. Note that agents in the model fully supply their labor, so that their current income only depends on productivity. As persistent trend growth shocks have a gradual but permanent effect on the level of the income, uncertainty regarding future income is very large. Households, facing high aggregate income risk, are more likely to save out of a precautionary saving motive, and this lowers the risk-free rate. By the same token, the economy with the deterministic productivity trend reduces the need for precautionary savings, raising the first moment of the risk-free rate.

4.2 The equity premium

Under the assumption of a deterministic trend, it seems that whether productivity level shocks ($\epsilon_{z,t}$) are permanent or persistent is crucial in explaining the risk premium. However, even the model that contains permanent level shocks along with the time deterministic trend (specification 4) cannot produce a large enough equity risk premium. Although specification 4 was shown in KL to be successful in generating the sizable equity premium to levered consumption claim, it fails here when productivity is directly estimated from the data.\textsuperscript{17} Nonetheless,

\textsuperscript{16} Notice that for LRR, the two effects cancel (so LRR does not affect the risk-free rate) when $\psi = 1$.\textsuperscript{17}
specification 4 seems to do better at generating the risk premium relative to specification 3. KL gave the key explanation for this. They argue that the more persistent technology shocks are, the more sizable the endogenous long-run consumption risk is, thereby generating a larger risk premium.

On the other hand, the process \( z_i \), under the stochastic base trend, seems to not make a significant difference in generating moments for asset returns. In particular, the models with specifications 1 and 2 do a good job of matching the data, though they fail to explain the second moment. At first glance, it seems to go against KL’s intuition that persistence of productivity shocks determines the size of the risk premium. However, as described in Subsection 4.3.2, when one considers persistent trend growth, the process \( z_i \) plays only a marginal role in generating the LRR in consumption. Hence, for the price implication, what matters is not the persistence of \( z_i \), but the existence of persistent trend growth.

### 4.3 The equity premium and GDP growth

Table 8 also presents correlations between the equity risk premium and macro variables such as GDP, investment, and consumption growth. In the data, the risk premium has a modestly positive association with GDP growth of 0.092. While this feature is properly replicated by specifications 1 and 2 where the correlation coefficients range from -0.001 to 0.084, specifications 3 and 4 generate correlations that seem to be near unity. I now examine a lead-lag structure between the risk premium and output growth in both the data and the model.

---

17) KL use \( \sigma_z = 4.11\% \), which is more than three times my estimate of 1.27\%, so that they obtain the annual return to levered consumption claim of 4.5\%. 

While it is widely accepted that equity returns lead output growth (Stock and Watson 1989; Cochrane 1991), the lead-lag relationship between the equity risk premium and output growth has been relatively less investigated.\(^{18}\) However, given that the equity return is more volatile than the risk-free rate, the lead-lag structure between the excess returns and output growth may be mainly driven by the excess returns. Figure 3 plots the cross-correlation between the equity risk premium and output growth. The length of the lags and leads is 12 quarters (i.e., three years) as in Gourio (2012). In the data, the equity risk premium positively leads output growth, and output growth negatively leads the equity risk premium; that is, as output growth is increased, the equity risk premium

\(^{18}\) Focusing on filtered GDP, Gourio (2012) examines this issue and found that the risk premium leads to filtered GDP.
decreases. Specifications 1 and 2 capture the fact that excess returns positively lead output growth, though they do less well with the negative lead of output growth when the number of lags is positive (i.e., the right side of this figure). On the other hand, specifications 3 and 4 achieve maximum correlation at zero lag and generate zero correlations for all other lags.

V. Conclusion

This paper explores the implications of various types of productivity processes in a production economy framework for macroeconomic quantities and asset prices. My findings suggest that compared to a deterministic trend, a stochastic productivity trend with persistent growth generates a larger LRR in consumption, and therefore, better explains several macroeconomic and asset pricing phenomena.

Four specifications for productivity are proposed and estimated using state space techniques from post-Korean War U.S. data. ML estimates show that productivity shocks are highly persistent in the deterministic trend case. On the other hand, in the presence of persistent trend growth, the persistence of the cycle shocks is estimated to be not statistically different from zero.

A limitation of the model is that it abstracts from stochastic volatility of the productivity process, which would be helpful in matching the data moments with a lower and more plausible coefficient of RRA. Fluctuating income uncertainty may lead to volatility risk, thereby resulting in higher second moments of asset returns.

A valuable extension to the model would be to consider leisure choice. Since it is known that hours of work are quite procyclical, allowing for leisure choice would increase the importance of the cycle component in productivity. It would be useful to test whether the same result holds in the framework where the labor supply decision is endogenous.
References


Appendix

A. Consumption predictability

In this section, I provide evidence for a persistent component of consumption growth by estimating the BY model using aggregate consumption data over a period from 1955:1 to 2014:4, 240 observations. The BY model under a homoscedasticity assumption is as follows:

\[ \Delta c_t = \mu + x_{t-1} + \epsilon_t, \]
\[ x_t = \rho x_{t-1} + \eta_t, \]
\[ \epsilon_t \sim N(0, \sigma^2_\epsilon), \quad \text{and} \quad \eta_t \sim N(0, \sigma^2_\eta), \]

where the two shocks \( \epsilon_t \) and \( \eta_t \) are mutually independent. \( \rho \) determines a persistent effect of \( \eta_t \) on consumption growth prospects (\( x_t \), the conditional expectation of demeaned consumption growth). As the BY model contains an unobserved state variable \( x_t \), I estimate the model using the Kalman filtering method.

<table>
<thead>
<tr>
<th>Table 9. Estimation results for U.S. consumption growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter estimates</td>
</tr>
<tr>
<td>( \rho )</td>
</tr>
<tr>
<td>----------------------</td>
</tr>
<tr>
<td>0.726</td>
</tr>
<tr>
<td>(0.107)</td>
</tr>
</tbody>
</table>

Notes: This table presents the maximum likelihood estimates of the parameters for the BY model. Standard errors are in parentheses for U.S. quarterly per capita real consumption growth. The data span the period from 1955:1 to 2014:4.

I find evidence for long-run consumption risk through persistent growth prospects for consumption. Table 9 reports estimation results for the model. The results indicate that the growth prospects are persistent and have low conditional volatility (\( \sigma_\eta \)) being statistically different from
zero. Thus, the growth prospects change slowly over time and their unconditional volatility is substantial. In specific, roughly 38% of consumption growth volatility arises from variation in this low frequency component. This substantial variation in the growth prospects is displayed graphically in figure 1.

B. Log-linearization results

This section offers several properties of log-linear approximation for EZ preferences. Taking the log of both sides in equation (3) gives

$$\log(1 - 1/\psi)\! \cdot \! v_{c_t} = \log(1 - \beta + \beta\exp((1-1/\psi)ce_t)).$$

(22)

I log-linearize equation (22) around $ce$, the unconditional mean of the certainty equivalent $ce_t$, to obtain equation (4),

$$v_{c_t} = \kappa(ce) + \beta'(ce)ce_t,$$

(4)

where

$$\kappa(ce) = \frac{1}{1-\psi^{-1}} \log(1 - \beta + \beta\exp(1-1/\psi)ce) - \beta'(ce)ce$$

and

$$\beta'(ce) = \frac{\beta\exp((1-1/\psi)ce)}{1 - \beta + \beta\exp((1-1/\psi)ce)}.$$ It is easy to verify that $\beta'(ce)$ is increasing in $ce$. Note that $\kappa(ce) \rightarrow 0$ and $\beta'(ce) \rightarrow \beta$ in the Tallarini (2000) world (i.e., when $\psi = 1$).

Now I derive an expression for the certainty equivalent. Assuming $\Delta c_t$ and $vc_t$ are log-normal and conditionally homoscedastic,

$$ce_t = (1-\gamma)^{-1} \log E_t \left[ \exp((1-\gamma)(vc_{t+1} + \Delta c_{t+1})) \right],$$

$$= E_t \left[ vc_{t+1} + \Delta c_{t+1} \right] - 0.5(\gamma - 1)^2$$

(23)

where $\sigma^2 \equiv Var_t[vc_{t+1} + \Delta c_{t+1}]$. Substituting into equation (4) gives
\[ vc_t = \kappa(ce) + \beta^*(ce)E_t[vc_{t+1} + \Delta c_{t+1}] - \beta^*(ce)0.5(\gamma-1)\sigma^2. \]

Iterating forward leads to

\[ vc_t = \frac{\kappa(ce)}{1-\beta^*(ce)} + \frac{(1-\gamma)\sigma^2}{2(1-\beta^*(ce))} + \sum_{j=1}^{\infty} [\beta^*(ce)]^j E_t[\Delta c_{t+j}]. \quad (24) \]

Inserting (24) into (23), we have in the end proposition 2:

\[ ce_t = \sum_{j=0}^{\infty} (\beta^*(ce))^j \left\{ \kappa(ce) - 0.5(\gamma-1)\sigma^2(ce) + E_t[\Delta c_{t+1+j}] \right\}. \]

Under the assumption that revisions in realized and future consumption growth are independent,

\[ \sigma^2 \equiv \Var_t[vc_{t+1} + \Delta c_{t+1}] \]
\[ = \Var_t \left[ \sum_{j=1}^{\infty} \beta^j(ce)E_{t+1}[\Delta c_{t+1+j}] \right] + \Var_t \left[ E_{t+1}[\Delta c_{t+1}] \right] \quad (25) \]
\[ = \Var_t \left[ \sum_{j=1}^{\infty} \beta^j(ce)(E_{t+1} - E_t)\Delta c_{t+1+j} \right] + \Var_t \left[ (E_{t+1} - E_t)\Delta c_{t+1} \right]. \]

We can rewrite equation (2) in terms of the log SDF as

\[ m_{t+1} \equiv \log\beta - (1/\psi)(\Delta c_{t+1}) + (1/\psi - \gamma)(vc_{t+1} + \Delta c_{t+1} - ce_t) \]
\[ = \log\beta - 0.5(1/\psi - \gamma)(1-\gamma)\sigma^2 - (1/\psi)\Delta c_{t+1} \]
\[ + (1/\psi - \gamma)(vc_{t+1} - E_t[vc_{t+1}] + \Delta c_{t+1} - E_t[\Delta c_{t+1}]). \quad (26) \]

Solving for \( vc_{t+1} - E_t[vc_{t+1}] \) from equation (24),

\[ vc_{t+1} - E_t[vc_{t+1}] = \sum_{j=1}^{\infty} \beta^j(ce)(E_{t+1} - E_t)\Delta c_{t+1+j}. \]

Substituting into equation (26) gives
\[ m_{t+1} = \log \beta - 0.5(1/\psi - \gamma)(1-\gamma)\sigma^2 - (1/\psi) \Delta c_{t+1} + (1/\psi - \gamma) \sum_{j=0}^{\infty} \beta^j (ce)(E_{t+1} - E_t)[\Delta c_{t+1+j}]. \] (27)

Hence, the revisions in the log SDF in proposition 3 is obtained as follows:

\[ (E_{t+1} - E_t)m_{t+1} = -\gamma(E_{t+1} - E_t)\Delta c_{t+1} - \left( \frac{1}{\psi} \right) \sum_{j=1}^{\infty} (\beta' (ce))^j (E_{t+1} - E_t) \Delta c_{t+1+j}. \]

The risk free rate is

\[ r_{f,t} = -E_t[m_{t+1}] - 0.5 \text{Var}_t[m_{t+1}] \]

\[ = -\log \beta + 0.5(1/\psi - \gamma)(1-\gamma)(SRR + LRR) + (1/\psi) \Delta c_{t+1} - 0.5\gamma^2 SRR - 0.5(1/\psi - \gamma) LRR, \]

where \( LRR = \text{Var}_t \left[ \sum_{j=1}^{\infty} \beta^j (ce)(E_{t+1} - E_t)[\Delta c_{t+1+j}] \right] \) and \( SRR = \text{Var}_t \left[ (E_{t+1} - E_t)[\Delta c_{t+1}] \right] \). It is convenient to rewrite in terms of LRR and SRR in order to see their effects,

\[ r_{f,t} = -\log \beta + \frac{1}{\psi} \Delta c_{t+1} + 0.5 \left( \frac{1}{\psi} - \gamma \right)(1-\gamma) - \gamma^2 \right) SRR + 0.5 \left( \frac{1}{\psi} - \gamma \right)(1-\frac{1}{\psi}) LRR. \]

C. Identification of two shocks

I investigate the identification issue of the structural specifications 1 and 2. By rearranging (5) – (8), we can obtain the following reduced form for specification 1:

\[ (1 - \rho_z L)(1 - \rho_s L)(1 - L)a_t = (1 - \rho_z)(1 - \rho_s)\mu + (1 - \rho_z L)\epsilon_{s,t-1} + (1 - \rho_s L)(1 - L)\epsilon_{z,t}. \] (28)
Equation (28) can be equivalently represented by the ARIMA(2,1,2) model:

\[(1 - \rho_s L)(1 - \rho_s L)(1 - L)a_t = (1 - \rho_s)(1 - \rho_s)\mu + (1 - \theta_1 L - \theta_2 L^2)e_t, \quad (29)\]

where \(e_t\) is \(i.i.d.\ \mathcal{N}(0, \sigma^2_e)\). With the autocovariances of \((1 - \rho_s L)(1 - \rho_s L)(1 - L)a_t\) from equations (28) and (29), the following simultaneous equation can be derived:

\[
\begin{bmatrix}
1 + \rho_s^2 & 2(1 + \rho_s + \rho_s^2) \\
-\rho_s & -(1 + \rho_s)^2 \\
0 & \rho_s
\end{bmatrix}
\begin{bmatrix}
\sigma_s^2 \\
\sigma_s^2 \\
\sigma_s^2
\end{bmatrix}
= \begin{bmatrix}
1 + \theta_1^2 + \theta_2^2 \\
-\theta_1 + \theta_1 \theta_2 \\
-\theta_2
\end{bmatrix} \sigma_e^2. \quad (30)
\]

Equation (30) satisfies the order condition for identification because a set of two unknown parameters \((\sigma_s^2, \sigma_e^2)\) can be determined by the three equations. Allowing for their correlation makes them exactly identified (i.e., three unknowns and three equations), but I set the correlation to zero to make a direct comparison with specification 2.

Let’s now turn to specification 2. The logic remains the same. The reduced form representation for specification 2 and its corresponding ARIMA process can be described by equations (31) and (32), respectively:

\[(1 - \rho_s L)(1 - L)a_t = (1 - \rho_s)\mu + \epsilon_{x,t-1} + (1 - \rho_s L)\epsilon_{z,t} \quad (31)\]

and

\[(1 - \rho_s L)(1 - L)a_t = (1 - \rho_s)\mu + (1 - \theta L)e_t. \quad (32)\]

From equations (31) and (32), we can obtain the following simultaneous equation:

\[
\begin{bmatrix}
1 & 1 + \rho_s^2 \\
0 & -\rho_s
\end{bmatrix}
\begin{bmatrix}
\sigma_s^2 \\
\sigma_s^2
\end{bmatrix}
= \begin{bmatrix}
1 + \theta^2 \\
-\theta
\end{bmatrix} \sigma_e^2. \quad (33)
\]

Equation (33) implies that specification 2 is exactly identified.
D. Identification of an intercept

One may cast a question: why the intercept does not appear in equation (9). To answer the question, consider the data generating process including the intercept as follows:

\[ y_t = c + \mu t + \tau_0 + s_0 + \sum_{j=1}^{t-1} s_j + z_t, \quad \text{for } t = 1, \ldots, T, \tag{34} \]

where \( c \) is the intercept, and \( \tau_0 \) and \( s_0 \) are unobservable values for the trend and the stochastic drift at \( t = 0 \), all of which are fixed parameters. As \( s_t \) is a stationary process with zero mean, \( s_0 \) is set to zero, while \( \tau_0 \) is an additional parameter to be estimated because \( \tau_t \) is non-stationary. Given that \( \tau_0 \) is a parameter, \( c \) is no longer identifiable. Table 10 confirms this argument. I consider three cases: 1) restricting about \( c = 0 \), 2) restricting about \( \tau_0 = 0 \), and 3) unrestricted \( \tau_0 \) and \( c \). The second and third columns in the table show that, in the standard Kalman filter, \( \tau_0 \) performs the same role as the intercept. Therefore, once \( \tau_0 \) is considered as a parameter, \( c \) is redundant. The exact initial Kalman filter tells a similar story about \( c \). Including \( c \) makes the Hessian matrix singular because the elements corresponding to \( c \) are exactly zeros, and thus one cannot obtain a finite standard error regarding the estimated \( c \) that is the same as the starting value for ML estimation. In other words, the inclusion of \( c \) does not contribute to the achievement of ML at all.
Table 10. Intercept identification for specification 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Restricted $c = 0$</th>
<th>Restricted $\tau_0 = 0$</th>
<th>Restricted $\tau_0$ and $c$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
<td>Case 3</td>
</tr>
<tr>
<td>$c$</td>
<td>-8.5746 (0.0099)</td>
<td>-5.4709 (0.0000)</td>
<td></td>
</tr>
<tr>
<td>$\tau_0$</td>
<td>-8.5746 (0.0099)</td>
<td>-3.1037 (0.0000)</td>
<td></td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>0.6933 (0.0652)</td>
<td>0.6933 (0.0650)</td>
<td></td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>-0.0602 (0.3978)</td>
<td>-0.0602 (0.3821)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>0.0087 (0.0008)</td>
<td>0.0087 (0.0007)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.0026 (0.0011)</td>
<td>0.0026 (0.0011)</td>
<td></td>
</tr>
<tr>
<td>Log $L$</td>
<td>759.6938</td>
<td>759.6938</td>
<td>759.6938</td>
</tr>
</tbody>
</table>

Notes: ML estimates with standard errors in parentheses. The sample period is the first quarter of 1955 to the fourth quarter of 2014 with a total of 240 quarterly observations. Instead of the exact initial Kalman filter, the standard diffuse Kalman filtering is used to make the argument clear.

E. Data

Total population in the economy is Current Population Survey (CPS) civilian non-institutional population aged 16 to 64 (LNU00000000 - LNU00000097). Real aggregate consumption and investment data are from the NIPA quarterly data (Table 1.1.3), reported by the BEA. Per capital real consumption is the sum of nondurables and services, divided by the size of total population; per capita real investment is the ratio of nonresidential fixed investment to total population; and output is the sum of per capita real consumption and investment. The return data is from Ken French's webpage and deflated by the CPI (CPIAUCSL through FRED).

Productivity is measured as follows:

$$\log A_t = \log Y_t - \alpha \log K_t 1 - \alpha,$$

(35)
where $Y$ denotes per capita real output; $K$ is a measure of capital stock; and $\alpha$ is the share of capital income, which is set to 1/3. To construct the capital stock series, I use the perpetual inventory method. The capital stock series are obtained according to the following recursion:

$$K_t = (1-\delta)K_{t-1} + I_t,$$

where the depreciation rate $\delta$ is 0.02. The initial capital stock for the first quarter of 1948 is computed as the ratio of investment for the first quarter of 1948 to the sum of the depreciation rate $\delta$ and the quarterly average growth rate of investment for 1948:2–2014:4.

### 1. State space representation of specification 2

Specification 2 has the following state-space form:

$$a_t = Z\alpha_t,$$

$$\alpha_t = \tilde{\mu} + T\alpha_{t-1} + R\eta_t, \quad \eta_t \sim N(0,Q),$$

for $t = 1, \ldots, T$, where $Z = [1 \, \, 1 \, \, 0]$; $\alpha_t = [\tau_t \, z_t \, s_t]^T$; $\tilde{\mu} = [\mu \, 0 \, 0]^T$; and $\eta_t = [\epsilon_{z,t} \, \epsilon_{s,t}]^T$. Matrices $T$, $R$, and $Q$ are given by

$$T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & \rho_s \end{bmatrix}, \quad R = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}, \quad Q = \begin{bmatrix} 0 & 0 \\ 0 & \sigma^2 \end{bmatrix}.$$

Since $\tau_t$ and $z_t$ are non-stationary, the initial state vector $\alpha_0$ is partially diffuse. The initial state vector $\alpha_0$ is specified as

$$\alpha_0 = B_0\zeta_0 + R_0\eta_0, \quad \eta_0 \sim N(0,Q_0),$$

where $\zeta_0 \sim N(0,\kappa I_2)$ is a diffuse vector as $\kappa \to \infty$; $\kappa I_2$ is the unconditional
covariance matrix of the non-stationary initial state variable \( \zeta_0 \); and \( Q_0 \) is the unconditional covariance matrix of the stationary initial state variable \( s_0 \), which is \( \sigma_s^2(1-\rho_s^2)^{-1} \). The matrices \( B_0 \) and \( R_0 \) are given by

\[
B_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad R_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.
\]

\( \alpha_0 \sim N(0, P_0) \) and \( P_0 = \kappa P_\infty + P_* \), where \( P_\infty = B_0 B_0^T \) and \( P_* = R_0 Q_0 R_0^T \).

**G. Log-likelihood evaluation**

Given the initial values, \( \alpha_{1|0} = \bar{\mu} + TE[\alpha_0] \), \( P_{\infty,1|0} = TP_\infty T^T \), \( P_{*,1|0} = TP_* T^T + RQR^T \), and \( \log L_0 = 0 \), the exact initial Kalman filter and the standard Kalman filter update \( \alpha_{t|t-1} \) and \( P_{t|t-1} \) and, thereby, \( v_t \) and \( F_t \) for \( t = 1, \ldots, d \) and \( t = d+1, \ldots, T \), respectively, which enable the log-likelihood function to be calculated.

For \( t = 1, \ldots, d \),

\[
v_t = a_t - Z\alpha_{t|t-1},
\]

\[
F_{\infty,t} = ZP_{\infty,t|t-1}Z^T, \quad F_{*,t} = ZP_{*,t|t-1}Z^T
\]

\[
\log L_t = \log L_{t-1} - 0.5 \log 2\pi - 0.5 \log F_{\infty,t},
\]

\[
M_{\infty,t} = P_{\infty,t|t-1}Z^T, \quad M_{*,t} = P_{*,t|t-1}Z^T
\]

\[
F_t^{(1)} = F_{\infty,t}^{-1}, \quad F_t^{(2)} = -F_{\infty,t}^{-1} F_{*,t|t-1} F_{\infty,t}^{-1},
\]

\[
K_t^{(0)} = TM_{\infty,t} F_t^{(1)}, \quad K_t^{(1)} = TM_{*,t} F_t^{(1)} + TM_{\infty,t} F_t^{(2)},
\]

\[
L_t^{(0)} = T - K_t^{(0)} Z, \quad L_t^{(1)} = -K_t^{(1)} Z,
\]
\[ \alpha_{t+1|t} = \mu + T \alpha_{t|t-1} + K^{(0)}_t v_t, \]
\[ P_{\infty, t+1|t} = TP_{\infty, t+1|t} L^{(0)}_t T, \]
\[ P_{\star, t+1|t} = TP_{\infty, t+1|t} L^{(1)}_t T + TP_{\star, t+1|t} L^{(0)}_t T + RQR^T. \]

For \( t = d + 1, \ldots, T, \)
\[ v_t = a_t - Z \alpha_{t|t-1}, \]
\[ F_t = ZP_{t|t-1} Z^T, \]
\[ \log L_t = \log L_{t-1} - 0.5 \log 2\pi - 0.5 \left( \log F_t + v_t^T F_t^{-1} v_t \right), \]
\[ M_t = P_{t|t-1} Z^T, \]
\[ K_t = TM_t F_t^{-1}, \]
\[ L_t = T - K_t Z, \]
\[ \alpha_{t+1|t} = \mu + T \alpha_{t|t-1} + K_t v_t, \]
\[ P_{t+1|t} = TP_{t+1|t} L_t^T + RQR^T. \]

**H. Unconstrained specification**

We consider the unconstrained specification where the shocks to the trend growth component are correlated with the shocks to cycle component \( (E_t[\epsilon_s, \epsilon_z, \epsilon_t] = \rho_{sz} \sigma_s \sigma_z). \) I denote the model by specification 0. The results for the parameter estimates are reported in table 11. The log likelihood and AIC for specification 0 are somewhat larger than those for specification 1. The LR statistic for the zero correlation restriction (specification 1) is 2.115 with a bootstrap p-value of 0.097, which confirms the identification of the correlation between the two shocks.
The estimates for specification 0 are slightly different from specification 1 with respect to the persistence for and the magnitude of the cycle. They are large in specification 0, which indicates a positive correlation of the two shocks because of the two facts: 1) the observed variance of productivity growth is unchanged, and 2) a shock to the cycle always brings out a negative autocorrelation of productivity growth. This is confirmed by the estimated correlation of 0.999.

Despite the difference in the cycle estimates, the estimation result for specification 0 is in accordance with that for specification 1 with respect to fluctuations in trend growth. This argument is illustrated in figure 4, which shows growth in productivity, the estimated trend growth from specification 0 (trend growth 0), and the estimated trend growth from specification 1 (trend growth 1). As you can see, although fluctuations in trend growth 0 are smaller than in trend growth 1, trend growth 0 still has a lot to contribute to fluctuations in productivity growth.

### Table 11. Estimation results for the unconstrained specification.

<table>
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<tr>
<th>Parameter estimates</th>
<th>Goodness of fit</th>
<th>Likelihood ratio test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_s )</td>
<td>( \rho_z )</td>
<td>( \sigma_s \times 100 )</td>
</tr>
<tr>
<td>0.588</td>
<td>0.352</td>
<td>1.150</td>
</tr>
</tbody>
</table>

Notes: This table presents the maximum likelihood estimates of the parameters for specification 0, which is the specification 1 allowing for a correlation between trend and cycle shocks. The data span the period from 1995:1 to 2014:4. I use a bootstrap to estimate the sampling distribution of the likelihood ratio (LR) test statistics by generating data from the specification 1 (the zero-correlation restriction).

### 1. Numerical solution methodology

I exploit an envelope condition method (ECM) to solve the model numerically. In order to obtain the stationary solution to specification 1, I normalize all variables by \( e^{\mu + \tau_{-1}} \) and define \( \hat{X}_t \) as \( \hat{X}_t = X_t / e^{\mu + \tau_{-1}} \). Then the stationary version of the value function is given by:
Figure 4. Trend growth estimates for specifications 0 and 1

Notes: This figure plots productivity growth (dotted line), trend growth of specification 0 (solid line), and trend growth of specification 1 (dashed line). Specification 0 stands for the model where the trend shocks are correlated with the cycle shocks. I use the Kalman smoother to obtain the unobserved trend growth. Data spans the period from 1955:1–2014:4.

\[ W_t = \max \left( 1 - \beta \right) \hat{C}_t^{1 - \psi/(1 - \psi)} + \beta e^{(1 - \psi)/(1 - \psi)(\mu + s_{t-1})} (E_t \left[ W_{t+1}^{1 - \gamma} \right])^{(1 - \gamma)/1 - \psi} \],

where \( W_t = W(\hat{K}_t, s_{t-1}, s_t, z_t) \). The social planner’s maximization problem is subject to the stationary versions of the capital accumulation, the resource constraint, and the productivity process as follows:

\[
\begin{align*}
\hat{K}_{t+1} e^{(\mu + s_t)} &= \left( \phi \left( \hat{I}_t / \hat{K}_t \right) + 1 - \delta \right) \hat{K}_t, \\
\hat{Y}_t &= \hat{K}_t^\alpha e^{(1 - \alpha)(s_{t-1} + z_t)}, \\
\hat{Y}_t &= \hat{C}_t + \hat{I}_t, \\
s_t &= \rho_s s_{t-1} + \epsilon_s, \quad \epsilon_s \sim N(0, \sigma_s^2), \\
z_t &= \rho_z z_{t-1} + \epsilon_z, \quad \epsilon_z \sim N(0, \sigma_z^2).
\end{align*}
\]

I parameterize the value function with 3rd order ordinary polynomials.
As a solution domain, I use a uniformly spaced grid of $5 \times 5 \times 5 \times 5$ points for the state variables $\hat{K}_t$, $s_{t-1}$, $s_t$, and $z_t$. A 3-node Gauss-Hermite quadrature rule is used to approximate integrals.

In order to find a solution to the above Bellman equation, I iterate on value function using the ECM. Unlike the conventional value function iteration algorithm, the ECM enables us to solve for the policy function for consumption $(\hat{C}_t^*)$ using the envelope condition:

$$W_{K,t} = \frac{(1-\beta)\hat{C}_t^{s-1/\psi}}{\Phi_{K,t}} W_t^{1/\psi} \left[(1-\delta) + \Phi_t + \frac{\hat{K}_t e^{(1-\alpha)(s_{t-1}+z_t)} + \hat{C}_t^*}{\hat{K}_t}\right],$$

where the golden section search is used to find the optimal consumption. At the end of each fixed-point iteration, the coefficients of the polynomial are updated by a regression, and thus the value function is adjusted accordingly.
생산성의 추세성장 충격이 자산 가격에 미치는 영향 분석

이남강*

본고는 추세성장에 대한 충격의 지속성을 추정한 후, 생산경제(production economy)에 반복효용함수(recursive utility)를 고려한 모형을 이용하여 생산성 추세성장 충격의 지속성이 장기소비위험(long-run consumption risk) 채널을 통해 자산 가격에 미치는 영향을 분석하였다. 실증분석 결과, 추세성장에 대한 충격의 지속성을 발견할 수 있었으며, 생산성 프로세스가 추세성장에 대한 충격의 지속성을 포함한 모형이 시간추세와 순환(cycle) 혹은 임의보행(random walk)을 따르다는 전통적인 가정들에 비해 데이터를 잘 설명하는 것으로 나타났다. 시뮬레이션 결과는 장기소비위험의 크기를 결정하는 핵심 요인이 추세성장에 대한 충격의 지속성을 나타냈다. 따라서 시장에서 관찰되는 낮은 이자율과 높은 주식프리미엄(equity premium)을 생산경제모형을 통해 설명하기 위해서는 시간추세와 순환(cycle) 혹은 임의보행(random walk)을 따르는 생산성 프로세스가 아닌 추세성장에 대한 충격의 지속성을 고려해야 하는 것으로 나타났다.

핵심 주제어: 장기소비위험, 생산경제, 확률적 추세성장, 주식프리미엄

JEL Classification: E21, E23, E30, G12

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