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Vulnerable Growth: A Revisit

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Vulnerable Growth: A Revisit

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Vulnerable Growth: A Revisit

This paper studies the distributional linkages between future economic performance and current conditions by means of a flexible quantile regression method. The examination of the linkages suggests that the conditional quantiles are nonlinear, which offers a new perspective on the conditional distribution. The nonlinearity causes countercyclical volatility to break down in both the right and left tails, the breakdown being associated with positive skewness in the short-term. As a corollary, in periods of recessions accompanied by a financial crisis, downside risks inherent in the distribution are smaller than we would think otherwise based on linear quantile regression.

Keywords: D-vine Quantile Regression, Conditional Quantiles, Nonlinearity, Downside Risk

JEL Classification: C53, E32, E37, E44

Ⅰ. Introduction

It has been well documented that financial conditions provide meaningful information concerning fluctuations in future economic activity. Focusing on a point forecast, a large body of empirical work has examined and supported this link (see, among others, Stock and Watson, 2003). Recent works have extended the link to a probabilistic forecast. Relying on linear quantile regression, Adrian, Boyarchenko, and Giannone (2019), henceforth "ABG," document the ways in which financial conditions can predict the probability distribution of future GDP growth. The linear quantile regression method, however, is subject to a restrictive assumption on the shape of the conditional quantiles (Bernard and Czado, 2015). As a result, tail risks (both upside and downside risks) to GDP growth induced by linear quantile regression (Koenker and Bassett, 1978) may be inaccurate.

This paper uses D-vine based quantile regression (DVQR for short), introduced by Kraus and Czado (2017), to reexamine how current economic and financial conditions shape the distribution of future economic activity. The DVQR model is highly flexible in the sense that it makes no precise assumption about the shape of the conditional quantiles. Thus, it would be an appropriate tool to reinvestigate tail risks to future economic activity. The other point of departure from ABG is that I use the real GDP gap instead of real GDP growth to measure economic activity by means of Hamilton (2018)'s method, the regression-based detrending method, to capture the stationary relationship between current conditions and future economic performance.

Working with the U.S. GDP gap and the National Financial Condition Index (NFCI), I show that the GDP gap measuring economic conditions is more informative regarding shaping the conditional distribution of the future GDP gap at one-quarter ahead. Specifically, it provides useful information on both left and right tail risks to GDP as well as the median at one-quarter ahead. This is robust to the linear quantile regression method. The NFCI measuring financial conditions, meanwhile, is more informative about left tail risks to GDP in the medium term, at four-quarters ahead, which is consistent with ABG's result.

I demonstrate that the estimated conditional quantiles are nonlinear. The nonlinearity of conditional quantiles makes big differences in a couple of dimensions compared with the linear counterpart. One such dimension is conditional moments. Specifically, the linear quantile regression model predicts that worsening current conditions are linearly associated with both a decline in the median and a widening of the dispersion, thus showing a strongly negative correlation between them. Unlike the linear model, the DVQR model predicts that worsening conditions, which nonlinearly let the median descend, do not necessarily widen the dispersion. The dispersion is, without any doubt, negatively associated with the median in ordinary conditions but remains steady or is positively related to the median in the tails, i.e., outside the ordinary conditions. This leads to a condition-dependent relationship between the median and dispersion, which is reminiscent of the mean-variance frontier in the field of finance. The dispersion is the narrowest at roughly 1.5% of the median. A decrease in current conditions within the range of the median between -5% and 1.5% makes the lower quantiles fall more rapidly than the upper quantiles at one-quarter ahead. Such a feature gives rise to countercyclical volatility. Procyclical volatility occurs if the median lies elsewhere, but to a lesser degree than the countercyclical volatility.

The DVQR model also produces a systematic relationship between the dispersion and skewness. An important starting point of the conditional median at one-quarter ahead is 1.5% as well, at which the skewness value is zero. When the median is greater than the point, the widening dispersion is accompanied by positive skewness, indicating that upside risks (the dispersion in the right tail) outweigh downside risks (the dispersion in the left tail). Countercyclical volatility with negative skewness occurs when the median is within the range of -5% and 1.5%. If the median falls below -5%, the model predicts procyclical volatility with positive skewness.

The second dimension is downside risks, defined as the distance between the middle and lower quantiles, over the business cycle. I find that, compared to the DVQR model, the linear quantile regression model greatly overstates downside risks during recessions, especially during the recessions accompanied by a financial crisis. Intuitively, such overestimation of downside risks arises from the restrictive assumption of the linear model, which induces constant but possibly separate marginal contributions of financial conditions to both the middle and lower quantiles. Indeed, no matter what the NFCI is, worsening financial conditions always reduce the lower quantiles to a greater extent than the middle quantile. So, the linear model cannot take account of mean reversion in the GDP gap at the height of severe recessions, thereby increasing future downside risks. Unlike the linear model, the DVQR model shows that financial stress in extremely bad conditions has little to contribute to a further decline in the lower and upper quantiles, while they lower the median, indicating a likelihood of a reversal in the GDP gap with positive skewness.

These findings have important implications for economists and policy makers. First, analysis through the lens of the GDP gap, instead of GDP growth, indicates that financial conditions are the dominant driver of medium-term risks while economic conditions retain useful information about risks to the GDP gap in the short term. Therefore, when assessing policy actions that countervail threats to financial stability, policymakers should reckon with such a time lag even though they take quick action in the presence of imminent threats. Second, the linear model suggests the trade-off between the mean and variance of the GDP gap—reducing the gap comes with an increase in volatility. Contrary to conventional wisdom, this paper indicates that there is some room for central banks pursuing inflation targeting to be able to reduce the gap with little cost to volatility. Third, it would be more desirable for central banks to use DVQR than linear quantile regression when communicating downside risks to the future outlook, especially when the economy is near the bottom in terms of an economic downturn. This is because the overstated downside risks caused by the linear quantile regression model in bad times could put the economic recovery at risk by making firms nervous, therefore scaling back on investment.

This paper conducts an out-of-sample test and confirms that the in-sample results are stable. The mean forecasting performance of the DVQR model is an interesting dimension to be addressed. If the model accurately estimated the conditional quantiles, especially in the tails, it would improve the forecasting power of the conditional mean. Indeed, the DVQR model helps in predicting the conditional mean of the GDP gap relative to the simple regression model and the fitted skewed t-distribution to linear quantiles regression (as in ABG) at both one- and four-quarters ahead. The pseudo-out-of-sample predictions show that the DVQR model achieves the highest forecasting accuracy in terms of both mean absolute error (MAE) and root mean squared error (RMSE).

The remainder of the paper proceeds as follows. Section II presents the measures of financial conditions and the way of constructing the GDP gap as economic conditions. Section III presents the model, and Section IV presents the main empirical results and discusses their implications. Section V examines the out-of-sample test. Section VI concludes.

Ⅱ. Data and Construction of GDP Gap

This paper uses the quarterly series of the National Financial Conditions Index (NFCI, published by the Federal Reserve Bank of Chicago) as a proxy for financial conditions as in ABG. However, ABG's analysis is extended here to cover the period from 1971:Q1 to 2019:Q4. A zero value of the NFCI indicates a historical average of U.S. financial conditions. Values less than zero represent loose financial conditions relative to the historic average, while the positive values of the NFCI show tighter conditions than the average. Since it is constructed to have a standard deviation of one, it can be considered to be stationary.

A critical issue is a stationary measure of economic performance. Perhaps none has been as popular as real GDP growth, but it may not be appealing to capture the true relationship between current financial conditions and future economic performance. The U.S. economy has experienced a significant decline in the long-run growth rate of output (Antolin-Diaz, Drechsel, and

Petrella, 2017), which implies that the real growth rate contains a nonstationary component. To deal with this issue, this paper constructs the GDP gap by applying the Hamilton (2018) method to quarterly, seasonally adjusted, real GDP as available from the FRED database.[1] Hamilton extracts a stationary, cyclical component from observed nonstationary data by regression of y_{t+h} on a constant and the p most recent values of y as of date t. Then the residuals g_{t+h} are stationary for a broad class of underlying processes:

$$\hat{g}_{t+h} = y_{t+h} - \hat{\beta}_0 - \hat{\beta}_1 y_t - \hat{\beta}_2 y_{u-1} - \cdots - \hat{\beta}_p y_{t-(p-1)}.$$

I use $h = 8$ and $p = 4$ to consider the business cycle on the two-year horizon, as per Hamilton's recommendation. To obtain the GDP gap starting in 1971:Q1, this setup requires real GDP over the period from 1968:Q2 to 2019:Q4. Throughout the paper, I work with the GDP gap as a current economic condition and future economic performance.

Figure 1 plots the GDP gap and the NFCI over the period from 1971:Q1 to 2019:Q4. Troughs in the GDP gap correspond closely to the NBER chronology. However, as mentioned in Hamilton (2018), the GDP gap begins to decline before the NBER business cycle peak in every recession. Such a difference arises from a philosophical difference in determining the turning points of business cycles. The NBER follows the philosophy of Burns and Mitchell (1946), the "classical cycle," which defines a peak as a point in time when absolute levels of economic activity start to decline. The Hamilton method is based on the "growth cycle," a deviation around a trend. Specifically, the GDP gap derived from the method with $h = 8$ can be interpreted as the demeaned growth rates over a two-year period. Hence, the NBER-defined peak goes with any value of the GDP gap at which a positive value turns into a negative value.

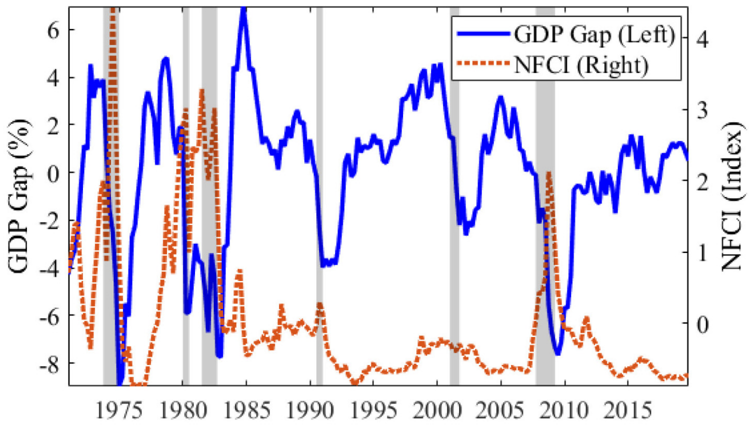
1) The Hamilton's regression-based method is more robust to real-time revisions than the Hodrick-Prescott (HP) filter that has been widely used to derive the gap (Jönsson, 2019).

Turning to the NFCI, the most striking feature is its high values in major periods of recessions: the recessions in the 1970s, 1980s, and the Great Recession of 2008 and 2009. These recessions were also severe in terms of economic performance measured as the GDP gap, which is consistent with empirical evidence that recessions accompanied by financial crises are more painful and longer than recessions not accompanied by financial crises (Claessens, Kose, and Terrones, 2012).

Ⅲ. The Model

This section describes the D-vine quantile regression model and its estimation (Kraus and Czado, 2017). Before doing so, I offer a brief overview of linear quantile regression. Let a response variable Y and predictor variables $X_{1,\cdots,}X_k$ $(k \geq 1)$ be continuous random variables. The conditional distribution of Y given $X = (X_{1,\cdots,}X_k)$ is denoted by $F_{Y|X}(y|x)$. Then, the conditional τth quantile function of the distribution of Y given X is defined as the inverse of the conditional distribution; that is, $Q_{Y|X}(\tau|x) = F_{Y|X}^{-1}(\tau|x)$. Hence, the

Figure 1. GDP Gap and NFCI



Notes: Data from the Federal Reserve Economic Data database (FRED, https://fred.stlouisfed.org/). The solid line depicts the real GDP gap, which is constructed by Hamilton (2018)'s method. The dotted line depicts the quarterly NFCI, which is calculated as the average of the available weekly observations. The shaded vertical bars denote the NBER-dated recessions.

conditional quantile function must be monotone in the probability index τ.

The linear quantile regression model assumes that the predicted conditional quantiles are affine functions of predictor variables

$$\widehat{Q}_{Y|X}(\tau|x) = \hat{\beta}_0(\tau) + \Sigma_{j=1}^{k}\hat{\beta}_j(\tau)x_j. \tag{1}$$

Denoting the quantile loss function by $L_\tau(u) = u(\tau - I(u<0))$, the coefficients $\hat{\beta}(\tau) = \left(\hat{\beta}_0(\tau), \hat{\beta}_1(\tau), \cdots, \hat{\beta}_k(\tau)\right)$ is chosen such that

$$\hat{\beta}(\tau) = \underset{\beta(\tau)\in\mathbb{R}^{k+1}}{\operatorname{argmin}} \mathrm{E}\left[L_\tau\left(y - \hat{\beta}_0(\tau) - \Sigma_{j=1}^{k}\hat{\beta}_j(\tau)x_j\right)\right],$$

where $\mathbf{E}$ is the expectation operator.

A well-known pitfall of linear quantile regression is that the monotone quantile ordering may be reversed; that is, $\widehat{Q}_{Y|X}(\tau_m|x) > \widehat{Q}_{Y|X}(\tau_n|x)$ for any $\tau_m < \tau_n$. This pitfall is called quantile crossing (see, among others, Bassett and Koenker, 1982). It occurs because regression parameters are heterogenous across quantiles. As suggested by (1), the slope parameters depend on τ, which causes quantiles at different values of τ to cross when X_j whose domain consists of real numbers is chosen to be sufficiently large or small. As a result, if the slope parameters depend on τ, then the conditional quantiles cannot be linear in x (Bernard and Czado, 2015). Suppose slope parameters are deterministic $(\hat{\beta}_j(\tau) = \hat{\beta}_j)$ in (1) to avoid the quantile crossing problem. Deterministic slope parameters imply that the conditional variance of Y given X. Thus, the linear quantile regression model with deterministic slopes is not appropriate to capture fluctuations in downside and upside risks to GDP, which is of main interest in this paper.

1. Copula-Based Conditional Quantiles

The copula-based conditional quantile function naturally satisfies monotonicity and may be appropriate for us to model the conditional quantiles that are nonlinear in x. Let a response variable Y and predictor variables $X_{1,\cdots,}X_k$ have univariate marginal distribution functions F_Y and $F_j(j=1,\cdots,k)$. According to the *probability integral transform* (PIT) theorem, the distribution functions of the response and predictor variables, $V:=F_Y(Y)$ and $U_j:=F_j(X_j)$, are uniformly distributed on [0,1]. From Sklar's theorem, the joint distribution of Y and $X_{1,\cdots,}X_k$ is now defined as

$$F(y,x_1,\cdots,x_k) = C(v,u_1,\cdots,u_k),$$

where C denotes a copula that is a $(k+1)$-dimensional distribution function on the hypercube $[0,1]^{k+1}$ with uniformly distributed margins, i,e., $C:[0,1]^{k+1}\to[0,1]$ (see, among others, Joe, 1997; Nelsen, 2007 for a formal definition and a detailed examination of copulas). When F and C are differentiable, we have the following joint density function

$$f(y,x_1,\cdots,x_k) = c(v,u_1,\cdots,u_k)f_Y(y)f_1(x_1)\cdots f_k(x_k),$$

where c denotes the copula density

$$c(v,u_1,\cdots,u_k) = \frac{\partial^k}{\partial v\partial u_1\cdots\partial u_k}C(v,u_1,\cdots,u_k).$$

The conditional distribution of Y given $X=(X_{1,\cdots,}X_k)$ can be represented as the conditional distribution of the PIT random variable V given $U=(U_{1,\cdots,}U_k)$ as follows:

$$F_{Y|X}(y|x)=C_{V|U}(v|u).$$

The conditional quantile function for $\tau\in(0,1)$ is then given by

$$Q_{Y|X}(\tau|x)=F_Y^{-1}\left(C_{V|U}^{-1}(\tau|u)\right),$$

where $C^{-1}_{V|U}(\tau|u)$ is the conditional copula quantile function conditioned on the PIT values of predictor variables.

2. The D-vine Based Quantile Regression Model

A D-vine is a way of constructing multivariate copulas using a cascade of bivariate copulas, so-called pair copulas (see Aas, Czado, Frigessi, and Bakken, 2009). A D-vine starts from choosing a specific order of the variables. Since we are interested in the conditional distribution of the future GDP gap as a function of the current GDP gap and NFCI, we only consider a three-dimensional D-vine with order $V - U_{l_1} - U_{l_2}$, where (l_1, l_2) is allowed to be an arbitrary permutation of (1, 2). Then in the first tree, the dependence of V and U_{l_1} and the dependence of U_{l_1} and U_{l_2} is respectively modeled using pair-copulas. In the second tree, the conditional dependence of V and U_{l_2} given U_{l_1} is modeled. This path structure leads to the copula density function that is factorized as

$$c(v, u_{l_1}, u_{l_2}) = c_{VU_{l_1}}(v, u_{l_1}) \cdot c_{U_{l_1}U_{l_2}}(u_{l_1}, u_{l_2}) \cdot c_{VU_{l_2};U_{l_1}}(h_{V|U_{l_1}}(v|u_{l_1}), h_{U_{l_2}|U_{l_1}}(u_{l_2}|u_{l_1}); u_{l_1}),$$

where $c_{VU_{l_2};U_{l_1}}$ denotes the copula density associated with the conditional distribution of $(V, U_{l_2})|U_{l_1} = u_{l_1}$. A common assumption when modelling D-vines is to assume that $c_{VU_{l_2};U_{l_1}}$ does not depend on u_{l_1}, which is the so-called simplifying assumption. The conditional distribution function, which is also called the h function in the context of pair copula construction (Aas, Czado, Frigessi, and Bakken, 2009), appearing as an element of $c_{VU_{l_2};U_{l_1}}$ can be obtained by partial differentiation (Joe, 1997):

$$h_{V|U_{l_1}}(v|u_{l_1}) := C_{V|U_{l_1}}(v|u_{l_1}) = \partial/\partial u_{l_1} C_{VU_{l_1}}(v, u_{l_1}) \text{ and}$$
$$h_{V|U_{l_1}}(u_{l_2}|u_{l_1}) := C_{U_{l_2}|U_{l_1}}(u_{l_2}|u_{l_1}) = \partial/\partial u_{l_1} C_{U_{l_2}U_{l_1}}(u_{l_2}, u_{l_1}).$$

Using a D-vine with order $V - U_{l_1} - U_{l_2}$, the conditional copula distribution $C_{V|U_{l_1}, U_{l_2}}$ can be expressed as

$$C_{V|U_{l_1}, U_{l_2}}(v|u_{l_1}, u_{l_2}) = h_{V|U_{l_2}; U_{l_1}}\left(h_{V|U_{l_1}}(v|u_1)|h_{U_{l_2}|U_{l_1}}(u_2|u_1)\right),$$

and its inversion leads to the conditional copula quantile function for $\tau \in (0,1)$:

$$C^{-1}_{V|U_{l_1}, U_{l_2}}(\tau|u_{l_1}, u_{l_2}) = h^{-1}_{V|U_{l_1}}(h^{-1}_{V|U_{l_2}; U_{l_1}}(\tau|h_{U_{l_2}|U_{l_1}}(u_2|u_1))|u_1).$$

Hence, the D-vine based conditional quantile function becomes

$$Q_{Y|X_1, X2}(\tau|x_1, x_2) = F_Y^{-1}\left(C^{-1}_{V|U_{l_1}, U_{l_2}}\left(\tau|u_{l_1}, u_{l_2}\right)\right).$$

3. Estimation

To obtain an estimate of the conditional quantile function, we need to estimate 1) the marginal distribution functions F_Y, F_1, and F_2, and 2) the D-vine copula: the ordering $l = (l_1, l_2)$ and three bivariate copula functions with corresponding parameters.

First, the marginal distribution functions are estimated in a nonparametric way. Given a sample $(Z_t)_{t=1}^T$ observed from a population with distribution F, the Parzen-Rosenblatt (see Parzen, 1962; Rosenblatt, 1956) kernel distribution estimator is defined as

$$\hat{F}(Z) = \frac{1}{T}\Sigma_{t=1}^T K\left(\frac{Z - Z_t}{h}\right),$$

where $K(u) = \int_{-\infty}^u k(t)$ with $k(\bullet)$ being a kernel function and $h > 0$ a bandwidth parameter. In this study, a Gaussian kernel is considered as a kernel function. When it comes to a bandwidth parameter, I use the well-known leave-$(2l+1)$-out cross validation method proposed by Hart and Vieu (1990), since variables of interest in this study are time dependent. Pseudo copula data $(\hat{v},\hat{u}_1,\hat{u}_2)=(\hat{v}_t,\hat{u}_{1,t},\hat{u}_{2,t})_{t=1}^{T}$ is then obtained by transforming raw data $(y_t,x_{1,t},x_{2,t})_{t=1}^{T}$ with the help of the kernel estimator.

Second, following Kraus and Czado (2017), the D-vine is estimated by maximizing the conditional log-likelihood. Because the ordering affects the conditional log-likelihood, it is sequentially determined by choosing the most influential covariates. For example, in the first step, we compare the conditional log-likelihood for each of the pairs $(V,U_j)_{j=1,2}$, given a copula function, and then choose a covariate that yields the highest value of the conditional log-likelihood. If U_2 is chosen, the ordering $\hat{l}_1=2$. In the second step, for the remaining variable U_1, the D-vine with order $V-U_2-U_1$ is compared with order $V-U_2$. If the addition of U_1 improves a measure of goodness of fit, the D-vine is updated with order $V-U_2-U_1(\hat{l}_2=1)$. If not, the algorithm stops and returns the D-vine with order $V-U_2$.

Given copula data $(\hat{v},\hat{u}_1,\hat{u}_2)$ the conditional log-likelihood of an estimated D-vine copula with ordering $\hat{l}$, estimated parametric pair-copula families $\hat{F}$, and corresponding copula parameters $\hat{\theta}$ is defined as

$$\mathrm{cll}(\hat{l},\hat{F},\hat{\theta};\hat{v},\hat{u}_1,\hat{u}_2)=\textstyle\sum_{t=1}^{T} c_{VU_{l_1}}(v_t,u_{l_1,t};\hat{F}_{VU_{l_1}}\hat{\theta}_{VU_{l_1}})$$
$$+c_{VU_{l2}:U_{l1}}(C_{V|U_1}(v_t|u_{l_1,t};\hat{F}_{VU_{l1}}.\hat{\theta}_{VU_{l1}}),C_{U_2|U_1}(u_{l_2,t}|u_{l_1,t};\hat{F}_{U_{l2}U_{l1}},\hat{\theta}_{U_{l2}U_{l1}});\hat{F}_{VU_{l2};U_{l1}},\hat{\theta}_{VU_{l2};U_{l1}}).$$

The AIC-corrected conditional log-likelihood $(\mathrm{cll}^{\mathrm{AIC}})$ is also considered as measures of the model's fit:

$$\mathrm{cll}^{\mathrm{AIC}}(\hat{l},\hat{F},\hat{\theta};\hat{v},\hat{u}_1,\hat{u}_2)=-\mathrm{cll}(\hat{l},\hat{F},\hat{\theta};\hat{v},\hat{u}_1,\hat{u}_2)+2|\hat{\theta}|,$$

where $|\hat{\theta}|$ denotes the number of estimated parameters. The possible bivariate copula families considered in this study include Gaussian, t, Frank, Gumbel, Clayton, Crowder, and Joe-Clayton (BB7).

4. Conditional Tail Independence

Tail dependence gives an account of how a pair of random variables move closely in the tail of a bivariate distribution. It is generally investigated by means of the lower and upper tail dependence coefficients:

$$\lambda_L = \lim_{u \to 0^+} P\left(Y < F_Y^{-1}(u) | X < F_X^{-1}(u)\right) = \lim_{u \to 0^+} \frac{C(u,u)}{u} \text{ and}$$

$$\lambda_U = \lim_{u \to 1^-} P\left(Y > F_Y^{-1}(u) | X > F_X^{-1}(u)\right) = \lim_{u \to 1^-} \frac{1 - 2u + C(u,u)}{1-u}.$$

These measures hold for any exchangeable copula (i.e., $C(v,u) = C(u,v)$ for all v and u). However, this generally used definition may give rise to a misleading conclusion about conditional tail independence (Bernard and Czado, 2015). The conditional tail independence describes a situation where conditional quantiles are flat in the tail. Right conditional tail independence (RCTI) and left conditional tail independence (LCTI) are formally defined as follows:

$$\forall \tau \in (0,1), \lim_{x \to +\infty} F_{Y|X}^{-1}(\tau) = a(\tau) \text{ and } \lim_{x \to -\infty} F_{Y|X}^{-1}(\tau) = a(\tau).$$

For example, suppose that random variables Y and X with normal marginal distributions have the Gaussian copula as their dependence structure. Even though the tail dependence coefficients are zero ($\lambda_{L=0}$ and $\lambda_{U=0}$), Y is not conditionally independent of X in both tails as the conditional quantile depends on a predictor variable x:

$$F_{Y|X}^{-1}(\tau) = \mu_Y + \rho \frac{\sigma_Y}{\sigma_X}(x - \mu) + \sqrt{1-\rho^2}\, \sigma_Y \Phi^{-1}(\tau),$$

where $\Phi^{-1}(\tau)$ is the τ-quantile of a standard normal distribution.

To avoid the inconsistency, this paper uses the concept of intermediate tail dependence (Bernard and Czado, 2015). The lower and upper coefficients of intermediate tail dependence are written as

$$\overline{\lambda}_L = \lim_{u \to 0^+} \frac{2\log u}{\log C(u,u)} - 1 \text{ and } \overline{\lambda}_U = \lim_{u \to 1^-} \frac{2\log(1-u)}{\log(1-2u+C(u,u))} - 1,$$

where $\overline{\lambda}_L$ and $\overline{\lambda}_U$ takes values in [-1, 1]. For variables being conditionally independent in the tails, $\overline{\lambda}_L = \lambda_L = 0$ and $\overline{\lambda}_U = \lambda_U = 0$. Exchangeable pair copulas and the corresponding coefficients of (intermediate) tail dependence are summarized in Table 1.

Table 1. Selected Copulas and Tail Dependence Coefficients

	$C(u_1,u_2)$	λ_L	$\overline{\lambda}_L$	λ_U	$\overline{\lambda}_U$
Gaussian $-1 \le \rho \le 1$	$\Phi_\rho\left(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\right)$ where $\Phi_p(\bullet)$ is the cdf of a standard normal with ρ	0	ρ	0	ρ
Gumbel $\delta \ge 1$	$\exp\left(-\left((-\log u_1)^\delta + (-\log u_2)^\delta\right)^{\frac{1}{\delta}}\right)$	0	$2^{1-\frac{1}{\delta}} - 1$	$2-2^{\frac{1}{\delta}}$	1
Frank $\delta \in R \setminus \{0\}$	$-\delta^{-1}\log\left(1+\frac{(e^{-\delta u_1}-1)(e^{-\delta u_2}-1)}{e^{-\delta}-1}\right)$	0	0	0	0
Clayton $\delta > 0$	$\left(u_1^{-\delta} + u_2^{-\delta} - 1\right)^{-\frac{1}{\delta}}$	$2^{-\frac{1}{\delta}}$	1	0	0
Student t $-1 \le \rho \le 1$, $v > 0$	$t_{p,v}\left(t_v^{-1}(u_1), t_v^{-1}(u_2)\right)$ where $t_{p,v}v(\bullet)$ is the cdf of the Student t with a degree of freedom v and correlation ρ	$2t_{v+1}(k)$	1	$2t_{v+1}(k)$	1
Crowder (BB9) $\delta > 0,\ \theta \ge 1$	$\exp\left(-\left((\delta-\log u_1)^\theta + (\delta - \log u_2)^\theta - \delta^\theta\right)^{\frac{1}{\theta}} + \delta\right)$	0	$2^{1-\frac{1}{\theta}} - 1$	0	0
Joe-Clayton (BB7) $\delta > 0, \theta \ge 1$	$1-\left(1-\left(\left(1-(1-u_1)^\theta\right)^{-\delta} + \left(1-(1-u_2)^\theta\right)^{-\delta} - 1\right)^{\frac{1}{\delta}}\right)^{\frac{1}{\theta}}$	$2^{-\frac{1}{\delta}}$	1	$2-2^{\frac{1}{\theta}}$	1

Notes: For coefficients of tail dependence of Student t copula, $k = -\sqrt{1+v}\sqrt{1-\rho}/\sqrt{1+\rho}$. For more information on the tail dependence coefficients of bivariate copulas, please refer to Bernard and Czado (2015).

Table 2. Estimates of Model for Future GDP Gap

	One–quarter ahead			Four–quarter ahead		
	(1)	(2)	(3)	(4)	(5)	(6)
Copula family	Crowder	Crowder	Clayton	Crowder	Frank	Clayton
Order	x_{l_1}: GDP, x_{l_2}: NFCI	x_{l_1}: GDP	x_{l_1}: NFCI	x_{l_1}: NFCI, x_{l_2}: GDP	x_{l_1}: GDP	x_{l_1}: NFCI
$\hat{\delta}_{VU_{l1}}$	0.16 (0.08)	0.17 (0.09)	0.67 (0.12)	6.35 (6.87)	3.95 (0.52)	0.87 (0.13)
$\hat{\theta}_{VU_{l1}}$	5.30 (0.74)	5.21 (0.69)		8.49 (7.11)		
$\hat{\delta}_{VU_{l2};u_{l1}}$	20.00 (0.00)			0.07 (0.07)		
$\hat{\theta}_{VU_{l2};u_{l1}}$	7.42 (3.13)			1.58 (0.18)		
$\hat{\delta}_{U_{l2}u_{l1}}$	20.00 (0.11)			20.00 (0.06)		
$\hat{\theta}_{U_{l2}u_{l1}}$	23.93 (20.21)			14.52 (8.60)		
cll	174.99	163.70	21.70	51.71	28.41	30.67
AIC	−337.98	−323.40	−41.41	−91.42	−54.81	−59.34

Notes: The data are real GDP and the NFCI, quarterly, from 1971Q1–2019Q4. Standard errors are in parentheses.

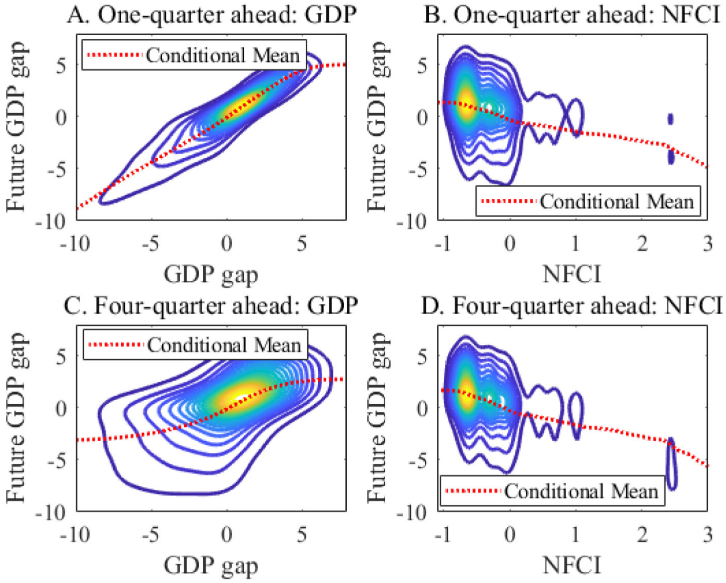
Ⅳ. Results

1. Parameter Estimates

Using the model discussed above, it is straightforward to examine 1) how current conditions shape the conditional distribution of the future GDP gap and 2) what types of conditions are more informative about the risks to the future gap at different time horizons. Table 2 shows the parameter estimates as well as measures of the model's fit. Estimates for the one- and four-quarter ahead GDP gap are respectively reported in columns (1)–(3) and (4)–(6). Columns (1) and (4) are baseline results for the model with both economic and financial conditions as predictor variables. Columns (2), (3), (5) and (6) show results for the model with a single predictor variable. Two noticeable features can be drawn from the table.

First, the distribution of the future GDP gap conditional on current economic and financial information features left tail risk. For both the one- and four-quarter ahead GDP gap, the estimated copula family is Crowder (a.k.a. BB9),[2] indicating

Figure 2. Contours of Estimated Density Functions



Notes: Panels A and B respectively depict the contours of the join density and the conditional mean specified by Crowder and Frank copula. Panels C and D show the contours of the join density specified by Clayton copula and the conditional mean.

intermediate dependence in the lower tails between future GDP and current conditions. The result is consistent with recent empirical evidence that the distribution of macroeconomic growth is negatively skewed (see, among others, Bekaert and Engstrom, 2017; Salgado, Guvenen, and Bloom, 2019).

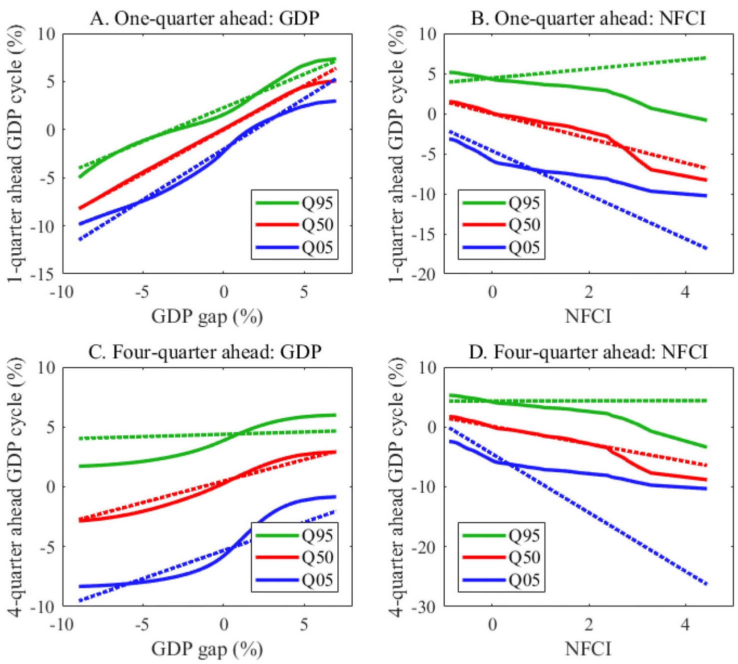
Second, at a one-quarter horizon, a current economic condition plays a more crucial role than a financial condition in describing left tail risk of the future GDP gap. This feature holds up to a three-quarter horizon. This result is shown in the Appendix. At a four-quarter horizon, however, the economic condition is less important than the financial condition. Column (5) shows that the selected type of copulas is Frank, which features conditional tail independence. Instead, the financial condition is the more important source of left tail risk in the four-quarter ahead GDP gap.

2) For the simplicity of estimation, this paper only considers D-vine copulas where all pair-copulas belong to the same family of copulas.

To better appreciate conditional tail dependence between the future GDP gap and current conditions, Figure 2 shows the contours of the joint density functions as well as the conditional mean of the future GDP gap. Clearly, the one-quarter ahead GDP gap has strong dependence, especially left tail dependence, on the current economic condition, while the four-quarter ahead GDP gap has rather weak dependence on the economic condition. A current economic condition is silent for right tail dependence. For both the one- and four-quarter ahead GDP gap, the current GDP gap that goes beyond 5.0% does not contribute to any change in the conditional distribution.

What does the second result from Table 2 imply in terms of macroeconomic modelling and policy design? A traditional macroeconomic model holds economic conditions to be more important than financial conditions. This approach can still be valid in predicting the evolving risks to future economic

Figure 3. Estimated Quantiles



Notes: The figure shows the univariate D-vine quantile regressions of the one-quarter ahead (top panels) and four-quarter ahead (bottom panels) GDP gap on current economic and financial conditions at five, 50 and 95 percent. The linear quantile regression lines (dotted lines) are superposed.

activities at least in the short-run (at one- to three-quarters ahead). However, macroeconomists should reckon with financial conditions to capture medium-term risks to future economic activities. Policy makers should take such a time lag into account when assessing or designing policy actions that offset threats to financial conditions.

2. Nonlinearity of Conditional Quantiles

Perhaps the easiest way to understand the conditional relationship between current conditions and the future GDP gap is to utilize univariate D-vine quantile regression. The estimated conditional quantiles of the one- and four-quarter ahead GDP gap are shown in Figure 3. Panels A and C illustrate the conditional quantiles on the current GDP gap, while Panels B and D are associated with the NFCI. Each panel shows the point estimates for the median as well as the lower (0.05) and upper (0.95) quantiles. For comparison, the estimates of linear quantile regression are represented as the dotted lines.

The figure shows that the conditional quantiles are nonlinearly dependent on current conditions. When the current GDP gap lies between roughly -5% and 1.5%, the lower quantile falls more rapidly than the upper quantile at one-quarter ahead as the current economic condition deteriorates. When the GDP gap is outside the range, the upper quantile exhibits stronger variation than the lower quantile (Panel A). Specifically, the upper quantile falls more rapidly than the lower quantile as the economic condition goes back down from its peak and reaches an average. At four-quarter ahead, when the GDP gap lies between -3% and 5%, variation in the GDP gap drives stronger variation in the lower quantiles than the upper quantiles, whereas both the lower and upper quantiles become flat elsewhere (Panel C). These results indicate that an economic condition is of usefulness in predicting tail outcomes of the future GDP gap inside the ranges. The nonlinearity is less pronounced in the middle quantile relative to the upper and lower quantiles. The median of the one-quarter ahead GDP gap is almost linear except that the current GDP gap lies above 5%.

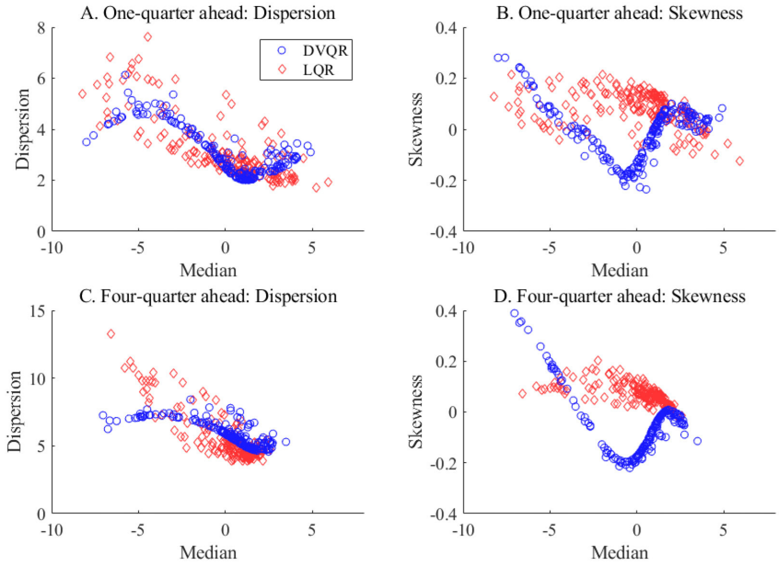
Turning to financial conditions, a tightening financial condition is associated with a decline in the middle and lower quantiles, whereas it is not connected with the upper quantile in the literature about the linear quantiles of the future GDP gap conditional on the NFCI. The DVQR model challenges this claim. The right panels of the figure show that the estimated quantiles depend nonlinearly on the financial condition. A tightening financial condition lowers the middle and lower quantiles, and reduces the upper quantile as well. A tightening financial condition has stronger effects on the lower quantile relative to the middle and upper quantiles only when a financial condition is looser than average, i.e., NFCI < 0. The nonlinearity stands out most in the middle quantile. As a financial condition tightens more than average, the middle quantile falls more rapidly than both the lower and upper quantiles. Specifically, when the NFCI goes from 2 to 3, the middle quantile plummets more than the other quantiles. This is robust at both one- and four-quarters ahead.

The nonlinearity of the conditional quantiles is the main finding and starting point of this article. In the next subsections, I examine its implications about the conditional distribution of the future GDP gap compared with the linear conditional quantiles.

3. Conditional Moments

How does the nonlinearity of the estimated conditional quantiles shape conditional moments of the future GDP gap as a function of both economic and financial conditions? To this end, I focus on the relationships between the conditional median, dispersion, and skewness of the one- and four-quarter ahead GDP gap, which are shown in Figure 4. The dispersion is measured by the interdecile range, the difference between the 90th and 10th percentiles. The skewness is measured by the Kelley's skewness, which is defined as the ratio of the difference between the dispersions in the right tail (the 90th percentile minus the median) and in the left tail (the median minus the 10th percentile) to the interdecile range.

Figure 4. Conditional Median, Dispersion, and Skewness



Notes: The figure shows scatter plots of the dispersion, as measured by the interdecile range, versus the median (panels A and C) and Kelley's measure of skewness versus the median (panels B and D). For comparison, the results from linear quantile regression (LQR) are reported as well.

The linear quantile regression model induces a strong, negative correlation between the median and dispersion, while there is no clear relationship between the median and skewness. This is because worsening economic and financial conditions are linearly related to both a decline in the median and a widening of the dispersion.

In contrast, the D-vine quantile regression yields that the dispersion and skewness are systematically associated with the median. For the one-quarter ahead GDP gap (Panels A and B), roughly 1.5% of the conditional median—at which the conditional dispersion is minimized while the Kelley skewness is zero—is a crucial starting point describing a picture of the conditional moments. Let's start with a familiar case where we can see counter-cyclical volatility. When the median lies between -5% and 1.5%, the dispersion sharply rises as the median falls, with negative values of the U-shaped skewness. This feature indicates that downside risks dominate upside risks. When the median is greater than 1.5%, the Kelley skewness is a concave function, with positive values, and the median is positively associated with the dispersion. Such a relationship implies a primary role for upside risks in shaping the conditional distribution of the GDP gap. However, the degree of such pro-cyclical volatility is smaller than that of counter-cyclical volatility within the range of -5% and 1.5% of the conditional median, showing the asymmetry. Another interesting case occurs when the conditional median goes beyond -5%. This case is characterized by positive values of the skewness and pro-cyclical dispersion. Given that the median being less than -5% is associated with the state of the economy being in the trough of the business cycle, the counter-cyclical skewness accompanied by the pro-cyclical dispersion reflects a strong possibility that the economy will recover quickly.

As depicted in Panels C and D of Figure 4, the conditional dispersion and skewness of the four-quarter ahead GDP gap show patterns similar to those in the one-quarter ahead GDP gap: the state-dependent cyclicality of the dispersion and skewness. A noticeable difference happens when the median is greater than 1.5%. Unlike the conditional skewness of the one-quarter ahead GDP gap, that of the four-quarter ahead GDP gap has negative values, which implies that downside risks grow with the prediction horizon.

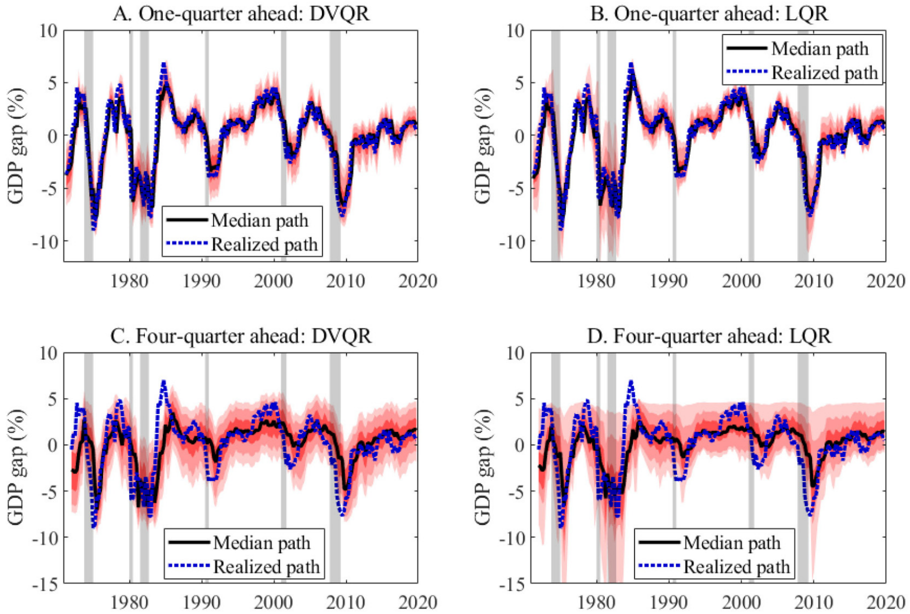
This finding has important implications for policy makers. The linear quantile regression model generates the trade-off between the mean and variance of the GDP gap. A decline in the gap is accompanied by a rise in volatility, which is commonly presumed.[3] Contrary to conventional wisdom, I argue that there may be room for policy makers to be able to reduce the gap with little cost to volatility, providing evidence for pro-cyclical volatility in an economic boom.

4. Growth-at-Risk Over Business Cycle

To better appreciate differences between the D-vine quantile function and the linear counterpart, I now investigate the evolution of the conditional

3) The negatively correlated mean and variance has been empirically supported (see, among others, Nakamura, Sergeyev, and Steinsson, 2017).

Figure 5. Predicted Distributions



Notes: The figure shows the predicted distribution, described by the fifth, 10th, 25th, 50th, 75th, 90th and 95th percentiles, of the one- and four-quarter ahead GDP gap over time. For comparison, the results from linear quantile regression (LQR) are shown as well.

distribution, focusing on growth-at-risk (GaR), defined as the GDP gap at the 5%, i.e., the 5th percentile, of the models over the business cycle. Figure 5 presents the estimated distribution, described by the fifth, 10th, 25th, 50th, 75th, 90th and 95th percentiles, of the one- and four-quarter GDP gap over time. The right panels (A and C) depict the D-vine quantiles, while the left panels (B and D) show the linear quantiles. This figure demonstrates that D-vine upper quantiles are not stable but vary significantly over time, which is consistent with Figure 4.

The most striking feature is that the linear quantile regression (LQR) model induces lower GaR than the D-vine quantile regression (DVQR) model during recessions accompanied by severely tough financial conditions. Those recessions include the 1973–1975 recession, the early 1980s recession, and the Great Recession of 2008 and 2009. The early 1990s and 2000s recessions were mild relative to the other recessions in the sense that they only

Table 3. Downside Risks over Business Cycles

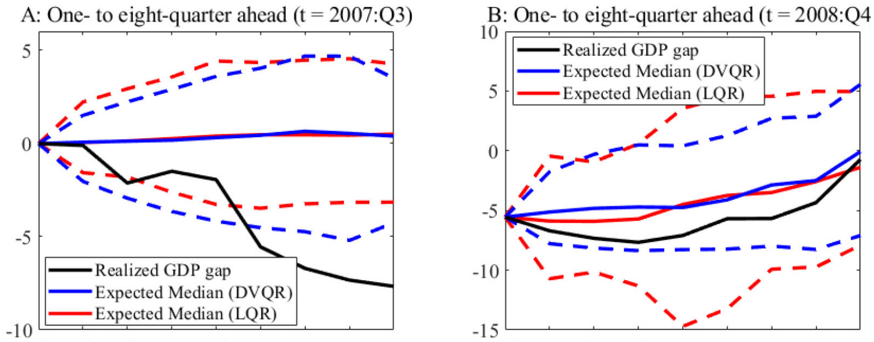
	Recessions				Expansion	
	Full	With financial crisis			Full	
	DVQR	LQR	DVQR	LQR	DVQR	LQR
	Panel A					
0.05 (GaR)	−5.58	−6.95	−6.42	−8.41	−1.42	−1.15
0.10	−4.99	−5.48	−5.84	−6.58	−0.93	−0.82
0.50	−2.98	−3.16	−3.73	−3.96	0.49	0.45
Median−GaR	2.60	3.79	2.69	4.45	1.91	1.60
	Panel B					
0.05 (GaR)	−7.03	−10.82	−7.81	−13.08	−3.46	−3.18
0.10	−6.20	−6.27	−7.08	−7.55	−2.32	−1.86
0.50	−2.93	−2.67	−3.85	−3.54	0.69	0.51
Median−GaR	4.10	8.15	3.96	9.54	4.15	3.69

interrupted the economy from keeping to grow and were not accompanied by any financially harsh problems.

Table 3 shows the downside risks, as measured by the dispersion in the left tail, i.e., the median minus GaR, alongside the middle and lower quantiles over the business cycle. First, compared to DVQR, LQR exaggerates the likelihood of severely adverse economic outcomes during recessions, especially during recessions accompanied by a financial crisis, while it underestimates the likelihood during expansions. Turning to the central tendency, LQR yields a lower median than its nonlinear counterpart during expansions. Consequently, LQR gives rise to a greater risk to the downside than the DVQR-induced downside risks during periods of recessions, whereas it makes for smaller downside risks than its nonlinear counterpart during expansions.

What type of model is more informative for policymakers who maintain vigilance against downside risks during periods of benign financial conditions and who plan for an economic recovery from recessions? To answer this question, I examine the global financial crisis by picking up two particular points in time: the third quarter of 2007 and the fourth quarter of 2008. The third quarter of 2007 was one quarter before the NBER-defined recession unfolded, which started in December 2017. Its corresponding NFCI was at -0.24, which started suddenly rising from -0.64 in the previous quarter.

Figure 6. Predicted Path over The Next Eight Quarters



Notes: The figure shows the paths of the predicted distribution of the GDP gap, described by the fifth, 50th and 95th percentiles, in the third quarter of 2007 (panel A) and the last quarter of 2008 (panel B) over the next eight quarters. For comparison, the results from linear quantile regression (LQR) are shown as well.

The fourth quarter of 2008 corresponds to the highest NFCI (2.13) during the crisis, which reflects heightened financial risks after the collapse of Lehman Brothers.

Figure 6 plots the paths of two predicted distributions conditional on the information available in the third quarter of 2007 (Panel A) and the last quarter of 2008 (Panel B) over one- to eight-quarter horizons. Panel A shows that DVQR practically provides more useful information regarding downside risks than LQR to policy makers who need to take preemptive action to tackle financial vulnerabilities and to reduce the potential likelihood of a financial crisis. Using information about economic and financial conditions in the third quarter of 2007, DVQR attaches a higher likelihood to extremely bad outcomes for the next two years by yielding a lower GaR than LQR. Notably, the left tail dispersion is wider than the right tail dispersion in DVQR for the next two years, indicating that downside risks are greater than upside risks. On the contrary, LQR generates upside risks that are greater than the downside risks for the next two years.

Turning to the last quarter of 2008 (Panel B), two quarters before the end of the recession, the DVQR and LQR models predict that in the future the economy would be on a recovery track. The models, however, make different predictions about tail risks. The DVQR model generates the path of a stable GaR and the path of the mildly increasing upper tail for the next two years, resulting in upside risks being larger than downside risks. On the other hand, the LQR model shows the GaR as being smaller than in the DVQR model, resulting in greater risks to the downside. Considering that central banks often communicate financial stability by reporting on the GaR, evaluating the GaR through the LQR model may dampen investors' appetite for new projects and consumer spending in the course of the recovery. Such evaluations and communication would delay any economic recovery.

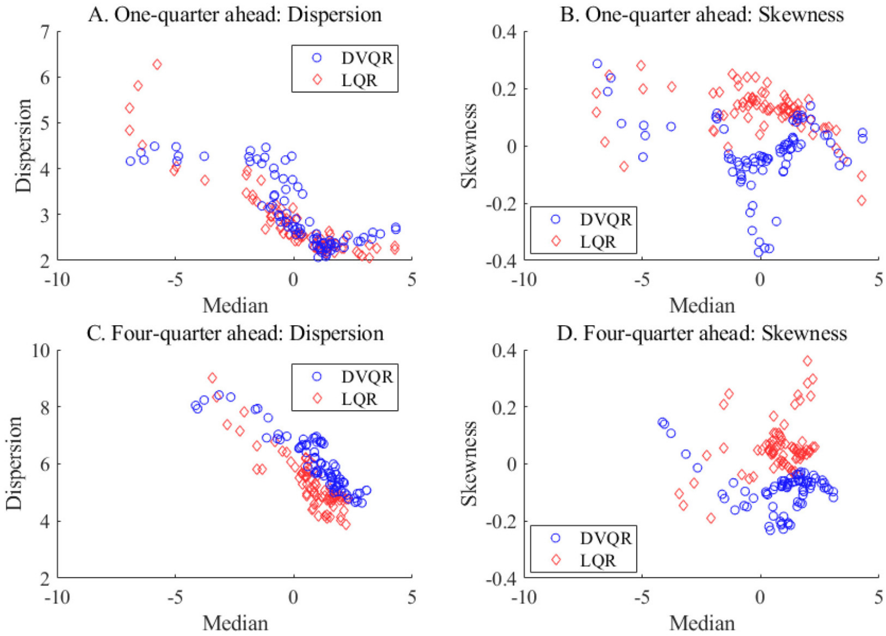
Ⅴ. Out-of-Sample Forecasts

In this section, I evaluate the out-of-sample performance to address the stability of the results from the previous section.[4] In the out-of-sample forecast, the starting date is held fixed, and the size of the in-sample window becomes larger as the ending date of the in-sample is advanced. Taking one-quarter ahead as an example, I first use data from 1971:Q1 to 1999:Q4 to estimate the predictive distribution for 2000:Q1. Then I update the in-sample from 1971:Q1 to 2000:Q1 to forecast the distribution for 2000:Q2. This procedure continues until I finish with an estimation using data from 1971:Q1 to 2019:Q3.

Conditional moments. Even using the out-of-sample prediction, Figure 7 shows that the out-of-sample results are virtually indistinguishable from the in-sample results. Both the dispersion and skewness seem to be systemically associated with the median. At one-quarter ahead, the dispersion begins to climb the right side of the median-dispersion curve as the economy is expected to continue to grow beyond a pivotal value, a positive GDP gap. The procyclical volatility coincides with positive skewness. Countercyclical volatility on the left

4) A caveat is that the out-of-sample analysis of the article is based on “pseudo-out-of-sample” forecasts in that the NFCI and the GDP gap are subject to revisions. However, the extent of revisions to both of them is not large enough to alter the results (Brave and Butters, 2012; Jönsson, 2019).

Figure 7. Out-of-Sample: Conditional Median, Dispersion, and Skewness

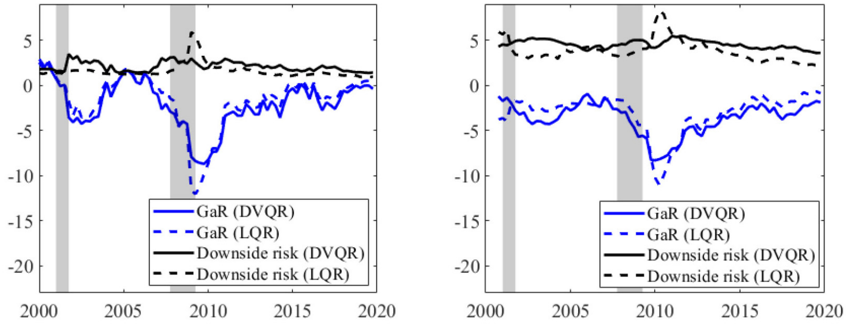


Notes: The figure shows the out-of-sample scatter plots of the dispersion, as measured by the interdecile range, versus the median (panels A and C) and Kelley's measure of skewness versus the median (panels B and D). For comparison, the results from linear quantile regression (LQR) are shown as well.

side of the curve is accompanied by negative skewness, which has been well documented. The left side also depicts the economy on recovery track after experiencing a severe recession, with positive skewness.

Downside risks over business cycle. Figure 8 plots both the GaR and downside risks, as measured by the difference between the median and the GaR. As we discussed in the previous section, compared to DVQR, LQR exaggerates downside risks during periods of recessions, especially accompanied by a financial crisis, whereas it underestimates them in normal times. These results hold for the out-of-sample analysis, as shown in Figure 8. The figure also confirms that the DVQR model provides useful information to policy makers who need to take preemptive actions in advance of a recession that is accompanied by a financial crisis. Specifically, Panel A of the figure shows that the downside risks derived from the DVQR model started to climb from 2007, signaling a recession ahead, while the downside

Figure 8. Out-of-Sample: Downside Risks over Business Cycle



Notes: The figure shows the out-of-sample time plots of both the GaR and the downside risks, as measured by the distance between the median and the GaR.

risks in the LQR model show little variation.

Mean forecasts. I conclude this section by comparing mean forecast errors from the out-of-sample test of three models, including 1) the DVQR model, 2) the fitted LQR model to the skewed t-distribution (as in ABG 2019), and 3) the linear regression model with both economic and financial conditions as prediction variables. The LQR model is typically faced with a problem of quantile crossing, which motivates ABG to approximate the conditional quantiles to the skewed t-distribution in order to obtain a distribution function. On the other hand, the DVQR model avoids the issue by construction. Thus, we can directly draw a discretized distribution function. In this work, I approximate the distribution function using 99 quantile functions for which a probability is uniformly chosen from [0.01, 0.99], and then compute the mean. I report two forecast evaluation statistics: 1) the root mean squared error (RMSE); and 2) the mean absolute deviation (MAD). Table 4 presents the out-of-sample performance. Interestingly, the DVQR model slightly outperforms the other competitors at both one and four quarters ahead. This result implies

Table 4. Pseudo Out-of-Sample Forecast Errors

	One-quarter ahead		Four-quarter ahead	
	MAD	RMSE	MAD	RMSE
DVQR	0.7895	1.0229	1.5060	2.1096
LQR	0.7968	1.0343	1.5549	2.1812
OLS	0.7962	1.0285	1.5420	2.1855

that the plausible estimation of tail risks helps in improving forecast accuracy.

Ⅵ. Concluding Remarks

The work of ABG has aroused considerable interest in the conditional distribution of future economic activities, not just among economists but also among policy makers and practitioners. The DVQR model gives a fresh look toward the distribution. This paper argues that the nonlinearity of the conditional quantiles gives us access to further information concerning the distribution. Countercyclical volatility is an important feature of the distribution. Notwithstanding, it is not the whole picture of the relationship between the median and dispersion. In particular, the trade-off between them breaks down in current economic and financial conditions that correspond to both the right and left tails of the distribution. The breakdown of the trade-off is associated with positive skewness in the short-term. This finding, in turn, provides a different view of the size of the downside risks over the business cycle. This paper shows that, compared to DVQR, linear quantile regression, the traditional method, exaggerates the downside risks during recessions that are accompanied by a financial crisis. These results have implications for the stance of monetary policy and macroprudential policy.

There are many useful directions for future research. First, this paper does not consider a structural model that is needed to avoid the Lucas critique. Thus, it would be strongly valuable to identify what causes such nonlinearity in an explicit structural model. Second, although I have provided the distributional link between the future GDP gap and the current GDP gap and NFCI, there is much more to do here. For instance, which specific measures of economic and financial conditions—industrial production, employment, credit growth, term spreads, and so on—are more important in describing the conditional distribution? How does their importance vary across different horizons? These questions would likely help policy makers have a probable economic outlook and take relevant action.

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Appendix

Table. A-1 Linear Quantile Regression

A. One-quarter ahead						
Quantile to estimate	0.05		0.50		0.95	
Constant	−1.97***	−2.03***	−0.07	−0.00	2.36	2.27
GDP gap	0.91***	1.05***	0.85***	0.91***	0.72***	0.70***
NFCI	−1.74***		−0.51***		0.56	
Pseudo R−squared	0.68	0.56	0.61	0.58	0.43	0.42
B. Two-quarter ahead						
Quantile to estimate	0.05		0.50		0.95	
Constant	−2.28***	−3.58***	−0.09	0.14*	2.90	2.64
GDP gap	0.64***	1.07***	0.68***	0.74***	0.64***	0.61***
NFCI	−2.04***		−0.96***		−0.12	
Pseudo R−squared	0.56	0.30	0.44	0.40	0.25	0.25
C. Three-quarter ahead						
Quantile to estimate	0.05		0.50		0.95	
Constant	−3.15***	−4.75***	−0.04	0.34	3.49	3.68
GDP gap	0.60***	0.88***	0.53***	0.68***	0.42**	0.45***
NFCI	−2.27***		−1.27***		−0.27	
Pseudo R−squared	0.47	0.17	0.34	0.26	0.07	0.07
D. Four-quarter ahead						
Quantile to estimate	0.05		0.50		0.95	
Constant	−4.21***	−5.31***	0.08	0.48**	4.34***	4.39***
GDP gap	0.37	0.47**	0.32***	0.36***	0.04	0.04
NFCI	−3.97***		−1.32***		−2.28	
Pseudo R−squared	0.34	0.10	0.24	0.12	0.00	0.00
E. Five-quarter ahead						
Quantile to estimate	0.05		0.50		0.95	
Constant	−4.29***	−5.95***	0.12	0.69**	4.36***	4.39***
GDP gap	0.29	0.36	0.15***	0.26***	0.01	−0.01
NFCI	−3.45***		−1.43***		0.13	
Pseudo R−squared	0.29	0.04	0.20	0.06	0.00	0.00
F. Six-quarter ahead						
Quantile to estimate	0.05		0.50		0.95	
Constant	−3.91***	−6.18***	0.07	0.69***	4.43***	4.38***
GDP gap	−0.00	0.09	0.03	0.17***	−0.05	−0.02
NFCI	−2.83***		−1.61***		−0.06	
Pseudo R−squared	0.25	0.00	0.15	0.03	0.00	0.00
G. Seven-quarter ahead						
Quantile to estimate	0.05		0.50		0.95	
Constant	−4.00***	−6.26***	0.07	0.67***	4.43***	4.37***
GDP gap	−0.33	0.08	−0.11	0.07	−0.26*	−0.18
NFCI	−3.55***		−1.54***		−0.43	
Pseudo R−squared	0.18	0.00	0.11	0.01	0.05	0.02
H. Eight-quarter ahead						
Quantile to estimate	0.05		0.50		0.95	
Constant	−3.87***	−6.24***	0.15	0.69***	4.16***	4.21***
GDP gap	−0.41**	−0.26	−0.27**	−0.04	−0.27**	−0.22**
NFCI	−2.98**		−1.43***		−0.34	
Pseudo R−squared	0.11	0.01	0.09	0.00	0.06	0.05

Notes: The data are quarterly, 1971:Q1–2019:Q4. Bootstrap significance levels are denoted by the number of asterisks. *** Significant at the 1 percent level.

Table. A-2 D-vine Quantile Regression

H-quarter ahead	1	2	3	4	5	6	7	8
Copula family	Crowder	Crowder	Crowder	Crowder	Crowder	Clayton	Crowder	Student-t
Order	x_{l_1}: GDP x_{l_2}: NFCI	x_{l_1}: GDP x_{l_2}: NFCI	x_{l_1}: GDP x_{l_2}: NFCI	x_{l_1}: NFCI x_{l_2}: GDP	x_{l_1}: NFCI x_{l_2}: GDP	x_{l_1}: NFCI	x_{l_1}: NFCI	x_{l_1}: NFCI x_{l_2}: GDP
$\hat{\delta}_{VU_{l1}}$	0.16 (0.08)	0.10 (0.06)	0.06 (0.05)	6.35 (6.87)	5.75 (6.74)	0.80 (0.13)	2.44 (2.59)	0.66 (0.09)
$\hat{\theta}_{VU_{l1}}$	5.30 (0.74)	3.17 (0.43)	2.17 (0.24)	8.49 (7.11)	7.54 (6.52)		3.59 (1.98)	7.53 (6.15)
$\hat{\delta}_{VU_{l2};u_{l1}}$	20.00 (0.00)	20.00 (0.00)	20.00 (0.00)	0.07 (0.07)	0.06 (0.08)			−0.18 (0.18)
$\hat{\theta}_{VU_{l2};u_{l1}}$	7.42 (3.13)	13.75 (3.40)	18.37 (3.86)	1.58 (0.18)	1.28 (0.12)			37.64 (90.19)
$\hat{\delta}_{U_{l2}u_{l1}}$	20.00 (0.11)	20.00 (0.00)	19.99 (0.57)	20.00 (0.06)	19.99 (1.36)			−0.84 (0.35)
$\hat{\theta}_{U_{l2}u_{l1}}$	23.93 (20.21)	18.14 (8.03)	10.59 (4.74)	14.52 (8.60)	13.70 (11.20)			171.28 (218.79)
cll	174.99	116.63	80.64	51.71	36.05	25.48	19.79	26.78
AIC	−337.98	−211.26	−149.29	−91.42	−60.11	−48.96	−35.59	−41.57

<Abstract in Korean>

실물·금융상황과 미래 실물경기 간의 비선형성

이남강*

본 연구는 D-vine 분위회귀모형을 이용하여 현재의 실물·금융상황과 미래 실물경기 간의 분포적 관계를 분석한다. 이때, 분석자료는 1971.1/4~2019.4/4분기 미국 실질 GDP 와 금융상황지수(NFCI)를 사용한다. 분석 결과, 미래 실물경기의 조건부 분위는 현재의 실물·금융상황과 비선형적 관계를 갖는 것으로 나타났다. 비선형성은 두 가지 시사점을 제공한다. 첫째, 비선형성을 반영하지 못하는 선형 분위회귀모형을 이용할 경우 금융위기를 동반한 경기침체 국면에서 하방위험을 과대 추정하여 투자 및 소비심리를 더욱 위축시켜 경기회복을 방해하는 요인으로 작용할 수 있다. 둘째, 미래 실물경기의 조건부 분포로부터 도출한 기댓값과 불확실성 간의 관계가 항상 음의 상관관계를 보인다는 기존의 연구결과와 달리 비선형성을 반영할 경우, 경기침체가 심화된 상황에서는 양의 상관관계를 보여 미래의 경기회복 가능성을 포착하는 데 유용한 것으로 나타났다.

핵심 주제어: D-vine 분위회귀, 조건부 분위, 비선형성, 하방위험

JEL Classification: C53, E32, E37, E44

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본고 작성에 유익한 논평을 주신 배병호 거시경제연구실장, 장보성 박사, 장희창 박사, 김종민 교수, 원내 세미나 참석자 및 익명의 심사위원께 감사의 뜻을 표합니다. 이 연구내용은 집필자의 개인의견이며 한국은행의 공식견해와 무관합니다. 따라서 본 논문의 내용을 보고하거나 인용할 경우에는 집필자명을 반드시 명시하여 주시길 바랍니다.

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